ON CERTAIN NON-LINEAR OPERATORS AND PARTIAL DIFFERENTIAL EQUATIONS

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1. Introduction and summary. Consider a partial differential equation

$$\Phi\left(\frac{\partial^k u}{\partial t^k}, \frac{\partial^k u}{\partial t^{k-1}\partial y}, \cdots, u, y, t\right) = 0 \tag{1.1}$$

with boundary conditions of the type

$$\begin{aligned} &\left.\frac{\partial^{2i} u}{\partial y^{2i}}\right|_{y=0} = \left.\frac{\partial^{2i} u}{\partial y^{2i}}\right|_{y=\pi} = 0 \qquad (i = 0, 1, \cdots, j) ; \\ &\left.\frac{\partial^i u}{\partial t^i}\right|_{t=0} = f_i(y) , \qquad (i = 0, 1, \cdots, k) . \end{aligned} \tag{1.2}$$

By means of a Fourier sine-series expansion with respect to one of the independent variables, say y,

$$\begin{aligned} u(y, t) &= \sum_{n=1}^{\infty} X_n(t) \sin (ny) , \\ X_n(t) &= \frac{2}{\pi}\int_0^\pi u(y, t) \sin (ny) dy \end{aligned} \tag{1.3}$$

there corresponds to the system (1.1), (1.2) an infinite system of ordinary differential equations in the X_n's

$$\Phi_n\left(t, X_1(t), \frac{dX_1}{dt}, \cdots, \frac{d^k X_1}{dt^k}, X_2(t), \cdots\right) = 0 \tag{1.4}$$

with the boundary conditions

$$\left.\frac{d^i X_n}{dt^i}\right|_{t=0} = \frac{2}{\pi}\int_0^\pi f(y) \sin (ny) dy \tag{1.5}$$

where

$$\begin{aligned} \Phi_n(t, s_1^0, s_1^1, \cdots, s_1^k, s_2^0, \cdots) = \frac{2}{\pi}\int_0^\pi \Phi\Big(\sum_{i=1}^{\infty} s_i^k \sin (iy), \\ \sum_{i=1}^{\infty} i s_i^{k-1} \cos (iy), \cdots, \sum_{i=1}^{\infty} s_i^0 \sin (iy), y, t\Big) \sin (ny) dy . \end{aligned} \tag{1.6}$$

Disregarding for the moment all questions of convergence of the series and permissibility of term by term differentiation and integration, the two systems (1.1), (1.2) and (1.4), (1.5) are equivalent; so that a

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