MULTIPLIERS OF COMMUTATIVE BANACH ALGEBRAS

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1. Introduction. In this paper Banach algebras will always mean complex commutative Banach algebras, with or without a unit. The concept of multipliers of a Banach algebra was introduced by Helgason [5] as follows: Let A be a semisimple Banach algebra considered as an algebra of continuous functions over its regular maximal ideal space X. Then by a *multiplier* of A is meant a function g over X such that $gA \subset A$. Every multiplier turns out to be a bounded continuous function, and the set of all multipliers of A under pointwise operations forms an algebra $M(A)$, called the *multiplier algebra* of A. In particular, if A is the algebra of all continuous functions on a locally compact Hausdorff space X which vanish at infinity under pointwise operations and supremum norm, then $M(A)$ is the algebra of all bounded continuous functions on X. In this case, Buck [2] introduced a topology on $M(A)$, called the *strict topology*, with many nice properties. In § 3, we will see that certain of Buck's results can be generalized to an arbitrary semisimple Banach algebra A.

The multiplier algebra can also be defined for a more general Banach algebra, and the strict topology can also be introduced in such a general case. § 2 will be devoted to discussions in such generality.

Next we narrow down to the case where A is a supremum norm algebra. In this case there are three natural topologies on $M(A)$, viz. the norm topology, the strict topology and the topology of uniform convergence on compact subsets of the maximal ideal space of A. It seems natural to ask when do two of these three topologies coincide. In § 4 we seek such characterizations in terms of topological properties of the Šilov boundary of A. Other problems regarding supremum norm algebras will be discussed in § 5.

Finally we will identify the multiplier algebras of certain Banach algebras which arise in harmonic analysis. Let S be an additive semigroup of positive integers and let A be the algebra of all continuous functions on the unit circle whose Fourier series involve e^{inx} with $n \in S$ only. Let $M(S)$ be the set of all integers m such that $m + S \subset S$. By an application of Fejér's theorem on the Cesáro summability we can prove that $M(A)$ is the algebra of all continuous functions on the unit circle whose Fourier series involve e^{inx} with $m \in M(S)$ only. In § 6 we will get a generalization of this result to arbitrary compact and even

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