AUTOMORPHISMS OF CLASSICAL LIE ALGEBRAS

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1. Introduction. Starting with a simple Lie algebra over the complex field C, Chevalley [2] has given a procedure for replacing C by an arbitrary field K. Under mild restrictions on the characteristic of K, the algebra so obtained is simple over its center, and it is our purpose here to determine the automorphisms of each such quotient algebra $\mathfrak{g}$. In terms of the group G defined in [2] and also in § 3 below and the group A of all automorphisms of $\mathfrak{g}$, the principal result is that, with some exceptions, which occur only at characteristic 2 or 3, A/G is isomorphic to the group of symmetries of the corresponding Schläfli diagram. As might be expected, the main step in the development is the proof of a suitable conjugacy theorem for Cartan subalgebras (4.1 and 7.1 below). The final result then quickly follows.

Definitions of the algebras and automorphisms to be considered are given in § 2 and § 3. Sections 4, 5 and 6 contain the main development and § 7 treats some special cases. The last section contains some remarks on the extension of the preceding results to other algebras. In 4.6, 4.7, 4.8, 7.2 and 7.3 the results are interpreted for the various types of algebras occurring in the Killing-Cartan classification, thereby yielding results of other authors [4, 5, 6, 7, 8, 9, 12, 14, 15, 16, 18] who have worked on various types of algebras from among those usually denoted A, B, C, D, G and F. For other treatments in which all types are considered simultaneously, the reader is referred to [4; 16, Exp. 16] where the problem is solved over the complex field, however by topological methods which can not be used for other fields, and to [14] where general fields occur but only partial results are obtained. General references to the classical theory of Lie algebras over the complex field are [1, thesis; 3; 16; 19].

2. The algebras. Let us start with a simple Lie algebra $\mathfrak{g}_C$ over the complex field C, a Cartan subalgebra $\mathfrak{h}_C$, the (ordered) system Σ of (nonzero) roots relative to $\mathfrak{h}_C$, the set Φ of fundamental positive roots, and for each pair of roots r and s, define c_{rs} to be the Cartan integer $2(r, s)/(s, s)$, and p_{rs} to be 0 if $r+s$ is not a root and otherwise to be the least positive integer p for which $r-ps$ is not a root. Then Chevalley [2, Th. 1] has shown that there exists a set of root elements $\{X_r\}$ and a set $\{H_r\}$ of elements of $\mathfrak{h}_C$ such that the equations of structure of $\mathfrak{g}_C$ are:

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