SOME CHARACTERIZATIONS OF A CLASS OF UNAVOIDABLE COMPACT SETS IN THE GAME OF BANACH AND MAZUR

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1. Introduction. The game of Banach and Mazur is understood here[1] as follows:

Two players A and B choose alternately nonnegative numbers t_n, $(n = 0, 1, 2, \cdots)$ in the following manner: B chooses a number t_0 such that $0 \leqq t_0 < 1$. After t_i $(i = 0, 1, \cdots, 2n)$ have been chosen, A chooses t_{2n+1} such that

$$\text{(a)} \qquad 0 < t_{2n+1} < t_{2n} \qquad (\text{if } t_0 = 0,\ t_1 \text{ is arbitrary})$$

and subsequently B a number t_{2n+2} such that

$$\text{(b}'\text{)} \qquad 0 < t_{2n+2} < t_{2n+1}\,, \qquad (n = 0, 1, 2, \cdots)\,.$$

Given a set $S \subset [0, 1]$, A will be said to win on S if $s = \sum_{n=0}^{\infty} t_n \in S$; otherwise B wins.

We shall deal in this paper with a generalization of this game, consisting in replacing (b′) by

$$\text{(b)} \qquad 0 < t_{2n+2} < k \cdot t_{2n+1}\,, \qquad (n = 0, 1, 2, \cdots)$$

where $k > 0$ will be referred to as the game constant.[2]

We say that the set S is unavoidable, or that B cannot avoid it, if there exists a sequence of functions $t_1(t_0), t_3(t_0, t_1, t_2), \cdots, t_{2n+1}(t_0, t_1, \cdots, t_{2n}), \cdots$, satisfying (a) and such that $s = \sum_{n=0}^{\infty} t_n \in S$ whenever (b) holds. If, on the other hand, there exists a sequence of functions t_0, $t_2(t_0, t_1), \cdots$, $t_{2n}(t_0, t_1, \cdots t_{2n-1}), \cdots$ satisfying (b) and such that $s = \sum_{n=0}^{\infty} t_n \notin S$, whenever (a) holds, then S is said to be avoidable.

The sets. In this paper we shall consider closed subsets of $[0, 1]$ exclusively. Let S be an arbitrary closed set on the interval $f = [0, 1]$

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[1] Various variants of the game are described in the so-called "Scottish Book", s. Coll. Math., **1** (1947), p. 57.

[2] The case of the constant k replaced by a variable k_n is considered in [**1**].