THE SOCHOCKI-PLEMELJ FORMULA FOR THE FUNCTIONS OF TWO COMPLEX VARIABLES

JERZY GORSKI

Introduction. In the case of one complex variable the following theorems are well known [3]:

1. Let C be a rectifiable oriented Jordan arc or curve and $f(\zeta)$ an integrable function defined on C, analytic at a point $z_0 \in C$ (in case C is an arc we suppose z_0 is different from both endpoints of C). Then the function

$$F(z) = \frac{1}{2\pi i}\int_C \frac{f(\zeta)}{\zeta - z}\, d\zeta$$

possesses the left and right limit $F_l(z_0)$ and $F_r(z_0)$, respectively, when the point ζ approaches to the point z_0 remaining permanently on one side of C and the relation

$$F_l(z_0) - F_r(z_0) = f(z_0)$$

holds.

2. Under the same conditions concerning the curve C suppose $f(\zeta)$ satisfies at every point $\zeta_1 \in C$ the Hölder condition

$$|f(\zeta) - f(\zeta_1)| \leqq M|\zeta - \zeta_1|^\alpha\,, \qquad M > 0\,, \qquad 0 < \alpha \leqq 1\,.$$

Then $F(z)$ possesses at almost every point $z_0 \in C$ the left and right limit when the point ζ approaches to z_0 along a non-tangent path to C and

$$F_l(z_0) = \frac{1}{2\pi i}\int_C \frac{f(\zeta)}{\zeta - z_0}\, d\zeta + \frac{1}{2} f(z_0)\,,$$
$$F_r(z_0) = \frac{1}{2\pi i}\int_C \frac{f(\zeta)}{\zeta - z_0}\, d\zeta - \frac{1}{2} f(z_0)\,.$$

The improper integral on the right hand side is taken in the Cauchy sense.

The aim of the present note is to extend these theorems to the theory of functions of two complex variables.[1] We start with Bergman's integral formula [1], [2] which generalizes the Cauchy formula for the case of functions of several variables. It would be very interesting to obtain similar results starting with other integral formulas which are similar to Bergman's formula e.g. A. Weil's formula [6] or later forms

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[1] Analogous results about the limits of exterior differential forms have been obtained by C. H. Look and T. D. Chung, see [4].