STRONGLY CONTINUOUS MARKOV PROCESSES

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Introduction. This paper is a continuation of [3]. We deal here with Markov processes with continuous parameter, while in [3] the discrete parameter case was studied. The notion of a "Markov Process" (here and in [3]) is different from the standard one: A stationary probability measure is assumed to exist, but the Chapman-Kolmogoroff Equation is replaced by a weaker condition. The exact definitions are given in § 1.

All problems are discussed from a Hilbert space point of view and convergence will mean, always, either strong of weak convergence.

1. Notation and background. We shall repeat here, for completeness, the notation of [3] and some of the results.

Let (Ω, Σ, μ) be a given measure space where $\mu(\Omega) = 1$, and $\mu \geqq 0$. The measure will be called the probability measure. The space of real square integrable functions is denoted by L_2.

Let $X_t(\omega)$ be a family of measurable real functions where $0 \leqq t < \infty$ and $\omega \in \Omega$. This will be called the Markov process and we assume:

If A is a Borel set on the real line and $t_1 < t_2 < t_3$ then the conditional probability that $X_{t_3} \in A$ given X_{t_1} and X_{t_2} is equal to the conditional probability that $X_{t3} \in A$ given X_{t_2}.

Also we assume that the process is stationary. Namely:

$$\mu(X_{t_1+s} \in A_1 \cap X_{t_2+s} \in A_2) = \mu(X_{t_1} \in A_1 \cap X_{t_2} \in A_2)$$

for all t_1, t_2, s positive real numbers and $A_1 A_2$ Borel sets.

For any set $\sigma \subset \Omega$, χ_σ denotes the characteristic function of this set. Let B_t be the closed subspace of L_2 generated by the functions $\chi_{X_t \in A}$. The self adjoint projection on B_t is denoted by E_t. Finally, let T_t be the transformation from B_0 to B_t defined by

$$T_t \chi_{X_0 \in A} = \chi_{X_t \in A}$$

where we used additivity to extend it to whole of B_0. In [3] the following equations are proved:

1.1 $$E_{t_1} E_{t_2} E_{t_3} = E_{t_1} E_{t_3} \qquad \text{if } t_1 < t_2 < t_3 .$$

1.2 a. $$\| T_t x \| = \| x \| , \qquad \text{for } x \in B_0 .$$

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