ON THE THEORY OF SPATIAL INVARIANTS

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1. Introduction. By an operator we mean in this paper a bounded linear transformation of a Hilbert space into itself. The adjoint of an operator T we will denote T^*. An operator is said to be normal if it commutes with its adjoint.

Given operators A_1, A_2 on the respective Hilbert spaces h_1, h_2 we say A_1 and A_2 are spatially equivalent if there exists a norm preserving linear transformation (called a spatial isometry) W of h_1 onto h_2 such that $WA_1W^{-1} = A_2$ on h_2, or equivalently, $W^{-1}A_2W = A_1$ on h_1. A major problem of operator theory is to determine a complete set of spatial invariants for an operator; two operators should be spatially equivalent if and only if they are assigned the same invariants.

This problem has been completely solved for the class of normal operators (see [5]). The weighted spectrum theorem for normal operators which generate maximal Abelian self adjoint algebras (henceforth denoted "masa") on separable Hilbert space can be stated as follows; any such operator T is determined within spatial equivalence by a family of equivalent finite Borel measures on the complex plane, concentrated on the spectrum of T, and this family is a spatial invariant (see bibliography of [5]). By the decomposition of Abelian W^*-algebras (see [5]) a certain sequence of such families constitutes a complete set of spatial invariants for an arbitrary normal operator on separable Hilbert space (whether generating a masa or not).

Let $\{A_n\}$ and $\{B_n\}$ be sequences of operators on the respective Hilbert spaces h and k. We say that $\{A_n\}$ is spatially equivalent $\{B_n\}$ if there exists an isometry W of h onto k such that $WA_nW^{-1} = B_n$ on k simultaneously for all n. Among other things we will find a complete set of spatial invariants for certain kinds of sequences of normal operators. And this we will apply to the theory of spatial invariants of some operators on separable Hilbert space which are not normal. Throughout this paper we will assume that all Hilbert spaces employed are separable in order that families of equivalent measures are at our disposal.

Each of the sections in this paper carries its own introduction and technical preliminaries. However an acquaintance with multiplication algebras of finite measure spaces, the weighted spectrum theorem on separable Hilbert space (cited above), and the decomposition of abelian W^*-algebras, is presupposed throughout.

2. Normal sequences. In this paper C will usually denote the com-

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