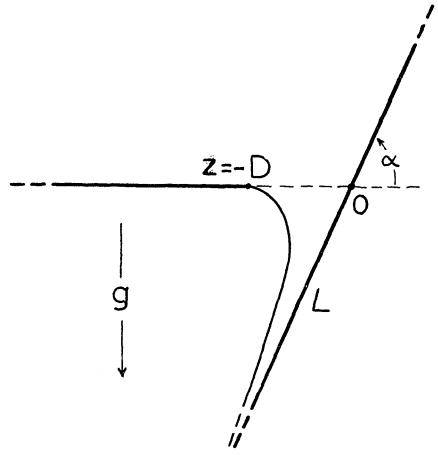
EXISTENCE OF A CLASS OF STEADY PLANE GRAVITY FLOWS

D. S. CARTER

1. Introduction. A number of exact solutions representing free boundary flows of an incompressible fluid under gravity appear in the literature (see [1] and references given there). As pointed out in [1], however, these are obtained by "inverse" methods, and exact theoretical treatment of problems in the large, having prescribed fixed boundaries and singularities, appears to have been confined to the case of periodic gravity waves.

In this paper we consider the family of steady plane irrotational flows of an incompressible inviscid fluid in a uniform gravitational field, with geometric configurations as illustrated in Figure 1. The fluid is



supported by a semi-infinite horizontal plane, and is bounded on the right by an infinite plane inclined at an angle α with the horizontal. The flow is downward through an open slot in the horizontal plane into a jet with a free boundary extending to infinity. This family includes the case of the symmetric jet from a slot, obtained when $\alpha = \pi/2$ by reflecting the flow across the vertical boundary.

In addition to the angle α, the physical parameters entering the problem include the constant specific force of gravity g, the slot width D, the fluid velocity at the slot edge q, and the total flow rate A (cross-sectional area of fluid entering the jet per unit time). Our principal result, contained in Theorem 1, § 3, and Theorem 4, § 11, is that

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