ON ESSENTIAL ABSOLUTE CONTINUITY

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Throughout this paper $\boldsymbol{D}$ will denote a bounded domain in Euclidean n-space R^n, and T will be a bounded, continuous, single-valued transformation from $\boldsymbol{D}$ into R^n. For such transformations, concepts of essential bounded variation and essential absolute continuity have been defined and studied by Rado and Reichelderfer ([3], IV. 4). In this paper a characterization of essential absolute continuity will be given. The characterization suggests a definition of uniform essential absolute continuity and some of the consequences of this definition will be investigated.

1. For every point x in R^n a multiplicity function $K(x, T, \boldsymbol{D})$ is defined ([3], II. 3.2). T is said to be essentially of bounded variation (briefly eBV) in $\boldsymbol{D}$ provided $K(x, T, \boldsymbol{D})$ is Lebesgue summable in R^n ([3], IV. 4.1, Definition 1). Let $X_\infty = X_\infty\ (T, \boldsymbol{D})$ denote the set of points x in R^n for which $K(x, T, \boldsymbol{D})$ is infinite. Thus if T is eBV in $\boldsymbol{D}$, then $\mathscr{L}X_\infty = 0$ (if A is a subset of R^n, then $\mathscr{L}A$ denotes its exterior Lebesgue measure). Since $K(x, T, \boldsymbol{D})$ is a lower semicontinuous function of x ([3], II. 3.2, Remark 10), X_∞ is a Borel set and, by Theorem 1 of [3], IV. 1.1, the set $T^{-1}\ X_\infty$ is also a Borel set.

2. If x is a point in R^n and C is a component of $T^{-1}x$ which is closed relative to R^n, then C is termed a maximal model continuum $(x, T, \boldsymbol{D})$ ([3], II. 3.1, Definition 1). Denote by $\mathfrak{C} = \mathfrak{C}(T, \boldsymbol{D})$ the class composed of all sets C for which TC is a point in R^n and C is a maximal model continuum for $(TC, T, \boldsymbol{D})$. Let $\mathfrak{E} = \mathfrak{E}(T, \boldsymbol{D})$ be the subset of $\mathfrak{C}$ consisting of those elements C each of which is an essential maximal model continuum (briefly e.m.m.c.) for $(TC, T, \boldsymbol{D})$ ([3], II. 3.3, Definition 1); the set $E = E(T, \boldsymbol{D}) = \cup\, C, C \in \mathfrak{E}$ ([3], II. 3.6). Let $\mathfrak{E}_i = \mathfrak{E}_i(T, \boldsymbol{D})$ be the subset of $\mathfrak{E}$ consisting of those elements C each of which is an essentially isolated e.m.m.c. (briefly e.i. e.m.m.c.) for $(TC, T, \boldsymbol{D})$ ([3], II. 3.3, Definition 2); the set $E_i = E_i(T, \boldsymbol{D}) = \cup\, C, C \in \mathfrak{E}_i$ ([3], II. 3.6.). Finally, let $\mathfrak{E}_i^p = \mathfrak{E}_i^p(T, \boldsymbol{D})$ be the subset of $\mathfrak{E}_i$ consisting of those elements of $\mathfrak{E}_i$ which consist of single points; the set $E_i^p = E_i^p(T, \boldsymbol{D}) = \cup\, C, C \in \mathfrak{E}_i^p$ ([3], II. 3.6). The sets E, E_i and E_i^p are Borel sets ([3], II. 3.6, Theorem 1).

If T is eBV in $\boldsymbol{D}$, then a necessary and sufficient condition that T be essentially absolutely continuous (briefly eAC) in $\boldsymbol{D}$ ([3], IV. 4.2) is

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