APPLICATIONS OF THE TOPOLOGICAL METHOD OF WAZEWSKI TO CERTAIN PROBLEMS OF ASYMPTOTIC BEHAVIOR IN ORDINARY DIFFERENTIAL EQUATIONS

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Introduction. The main objective of this paper is to present some results concerning the asymptotic behavior of the integrals of some systems of ordinary differential equations.

As Wazewski's theorem, used in our work, is not very well known, we state it here, giving first some definitions and notations.

HYPOTHESIS H. (a) *The real-valued functions* $f_i(t, x_1, \cdots, x_n)$, $i = 1, \cdots, n$, *of the real variables* $t, x_1, \cdots, x_n$, *are continuous in an open set* $\Omega \subset R^{n+1}$.

(b) *Through every point of* Ω *passes only one integral of the system*

$$\dot{x} = f(t, x) \quad \left(\cdot = \frac{d}{dt}\right) \quad \textit{where}$$

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}, \quad f(t, x) = \begin{pmatrix} f_1(t, x_1, \cdots, x_n) \\ \vdots \cdots\cdots\cdots\cdots \\ f_n(t, x_1, \cdots, x_n) \end{pmatrix} \qquad \textit{and } (t, x) \in \Omega .$$

Let ω be an open set of R^{n+1}, $\omega \subset \Omega$ and let us denote by $B(\omega, \Omega)$ the boundary of ω in Ω.

Let P_0: $(t_0, x_0) \in \Omega$. We write $I(t, P_0) = (t, x(t, P_0))$, where $x(t, P_0)$ is the integral of the system $\dot{x} = f(t, x)$ passing through the point P_0.

Let $(\alpha(P_0), \beta(P_0))$ be the maximal open interval in which the integral passing through P_0 exists. We write

$$I(\Delta, P_0) = \{(\mathrm{t}, x(t, P_0)) \mid t \in \Delta\}$$

for every set Δ contained in $(\alpha(P_0), \beta(P_0))$.

We say that the point P_0: $(t_0, x_0) \in B(\omega, \Omega)$ is a *point of egress* from ω (with respect to the system $\dot{x} = f(t, x)$ and the set Ω) if there exists a positive number δ such that $I([t_0 - \delta, t_0), P_0) \subset \omega$; P_0 is a *point of strict egress* from ω if P_0 is a point of egress and if there exists a positive number δ such that $I((t_0, t_0 + \delta], P_0) \subset \Omega - \overline{\omega}$. The set of all points of egress (strict egress) is denoted by $S(S^*)$.

If $A \subset B$ are any two sets of a topological space and $K: B \to A$ is

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