ON EXTREMAL PROPERTIES FOR ANNULAR RADIAL AND CIRCULAR SLIT MAPPINGS OF BORDERED RIEMANN SURFACES

PAUL A. NICKEL

Introduction. There exist functions which map a planar Riemann surface W of arbitrary conectivity conformally onto plane slit regions. Functionals I, extremized in the class of all conformal mappings of W by only one slit mapping, are known. Such functionals can be represented as limits of functionals I_n, where each I_n is itself extremized by a horizontal or vertical-slit mapping with domain of finite connectivity.

A planar bordered Riemann surface of finite connectivity can be mapped conformally onto a radial or circular-slit annulus with inner and outer boundaries corresponding to any two contours of the surface. In this investigation, extremal properties of such mappings are obtained and extended to surfaces of infinite connectivity. The geometric nature of the extended mappings, called principal analytic functions, is then deduced from the extended extremal properties. In addition, certain combinations of principal analytic functions are investigated from both extremal and geometric points of view.

First, we consider a planar bordered oriented Riemann surface $\bar{W}$, of infinite connectivity. It is assumed that $\bar{W}$ has two compact border components, δ and γ, such that no point of $\delta \cup \gamma$ is a limit point of points of any other boundary components. Such contours are called isolated. $\bar{W}$ is "approximated" by a sequence of compact bordered Riemann surfaces $\{W_n\}$, where each W_n is of finite connectivity. On W_n, annular radial and circular-slit mappings F_{0n} and F_{1n} are constructed. Among all normalized conformal annular mappings F of W_n, F_{0n} maximizes

$$2\pi \log(r(F)) + \mu_n(F)$$

and F_{1n} minimizes

$$2\pi \log(r(F)) - \mu_n(F) .$$

Here, $r(F)$ is the quotient r_γ/r_δ, where r_γ and r_δ represent the radii of the positively oriented $F(\gamma)$ and the negatively oriented $F(\delta)$ respectively, and $\mu_n(F)$ is the complementary area of $\log(F(W_n))$.

It is then shown by the reduction theorem (Sario[4]) that these extremal properties hold in the limit for the limit functions F_0 and F_1.

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