CATEGORY METHODS IN RECURSION THEORY[1,2]

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The heavy symbolism used in the theory of recursive functions has perhaps succeeded in alienating some mathematicians from this field, and also in making mathematicians who are in this field too embroiled in the details of thier notation to form as clear an overall picture of their work as is desirable.[3] In particular the study of degrees of recursive unsolvability by Kleene, Post, and their successors[4] has suffered greatly from this defect, so that there is considerable uncertainty even in the minds of those whose speciality is recursion theory as to what is superficial and what is deep in this area.[5] In this note we shall examine one particular theorem (namely the Kleene-Post theorem asserting the existence of incomparable degrees[6]) and show that it is a special case of a very easy and well-known theorem of set-theory. Exposition will be such as to require (except in a few footnotes) no preliminary acquaintance with recursive matters. It is to be hoped that some mathematicians in other areas may be stimulated by this exposition to try their hand at some open questions about recursive functions: it is to be hoped also that they will not carry away the impression that all of recursion theory is as trivial as this paper will show the Kleene-Post theorem to be.

First let me describe in an informal way what relative recursiveness is. The only properties of it which we shall need will be apparent from this informal discussion.

Denote by ε the set of all nonnegative integers. A *function* shall mean a number-theoretic function $f: \varepsilon \rightarrow \varepsilon$. A function is called *recursive* if it can be computed in an effective (mechanical) manner: we shall not need the details of the definition.[7] Sometimes two functions f and g are so related that the function f can be calculated in an effective

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[2] Category methods have also been used by the author in **[12]**, and form the basis of the entire treatment of degrees in **[3]**.

[3] A related (but much deeper) contribution to the methodology of recursion theory has made by Addison, e.g., in **[1]**.

[4] See, e.g., **[7]**, **[14]**, **[15]**, **[19]**. A sadly neglected paper in the same area which completely avoids these unnecessary complications is Lacombe **[10]**.

[5] The principal result of Spector **[19]** (minimal non-recursive degrees) is probably 'deep' in this sense, as is likewise the Friedberg-Mučnik proof (**[4]**, **[11]**) of the existence of incomparable degrees *of recursively enumerable sets*.

[6] Strictly speaking, the Kleene-Post theorem (**[7]**, p. 390) gives more information than our version, since it gives incomparable degrees $<\mathbf{0}'$. But this result too can be obtained by a category argument, as I shall show in a later publication.

[7] Cf., e.g., Davis **[2]**, p. 41.