AN IMBEDDING SPACE FOR SCHWARTZ DISTRIBUTIONS

DONALD E. MYERS

1. Introduction. We consider here a facet of the problem of justifying the methods of the operational calculus and in particular the use of the "Dirac Delta Function". L. Schwartz's "Theorie des Distributions" [6] is the most complete exposition to date on generalized functions but the operational calculus as such is largely omitted. B. Van Der Pol [8] discusses the latter but not in the context of distributions. Ketchum and Aboudi [4] suggested using unilateral Laplace Transforms to construct a link between Schwartz's theory and the operational calculus. This paper will enlarge on the latter suggestion. Two principal results are obtained. An imbedding space is constructed and a comparison between the topologies is made.

Let S denote the strip $\sigma_1 < R(z) < \sigma_2$, in the complex plane. Consider the one parameter family of functions $\{e^{zt}\}$, where the parameter z ranges over S and $-\infty < t < \infty$. This family is not a linear space but each member possesses derivatives of all orders. In a manner analogous to Schwartz we define an L_S-Distribution to be an analytic complex-valued functional on the above family of functions, where by analytic we mean with respect to the parameter z. If α is any complex scalar and F, σ are two such functionals then we require that $F \cdot e^{zt} + \sigma \cdot e^{zt} - (F + \sigma) \cdot \sigma^{zt}$, and $(\alpha F) \cdot e^{zt} = F \cdot (\alpha e^{zt})$. The latter property then allows us to define the derivative in a manner similar to that of Schwartz, that is $F' \cdot e^{zt} = F \cdot (e^{zt})' = F \cdot ze^{zt} = zF \cdot e^{zt}$. It also follows that the Laplace Transform supplies an integral representation of some of the functionals. The other L_S-Distributions define generalized functions for similar integral representations. That is, each function analytic for $z \in S$ has for its values, the values of an L_S-Distribution acting on a function e^{zt} and the L_S-Distribution has an integral representation utilizing the symbolic inverse Laplace Transform of the analytic function. In most of this paper we deal only with analytic functions whose inverse transforms exist but the definitions and theorems will be stated without this restriction where possible. Following a practice used by other authors, we will call the inverse Laplace Transform, symbolic or not, an L_S-Distribution rather than the functional. Because of the relation between the functional and an analytic function we concentrate on the latter and utilize the already known properties of such functions. By emphasizing the integral representations rather than the functionals we utilize the

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