ON N-HIGH SUBGROUPS OF ABELIAN GROUPS

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In a recent paper [2] the concept of high subgroups of Abelian groups was discussed. The purpose of this paper is to give further results concerning these high subgroups. All groups considered in this paper are Abelian, and our notation is essentially that of L. Fuchs in [1]. Let N be a subgroup of a group G. A subgroup H of G maximal with respect to disjointness from N will be called an *N-high* subgroup of G, or N-high in G. When $N = G^1$ (the subgroup of elements of infinite height in G), H will be called *high* in G.

After considering N-high subgroups in direct sums, we give a characterization (Theorem 3) of N-high subgroups of G in terms of a divisible hull of G. Next we show (Theorem 5) that if G is torsion, $N \subseteqq G^1$, and H is N-high in G, then H is pure and (Lemma 7) the primary components of any two N-high subgroups have the same Ulm invariants (see [3]). These results generalize the results in [2]. The concept of Σ-groups is introduced, and it is shown that any two high subgroups of torsion Σ-groups are isomorphic. Further, torsion Σ-groups are characterized in terms of their basic subgroups. Theorem 3 of [2] is generalized to show that high subgroups of arbitrary Abelian groups are pure. This leads to the solution of a more general version of Problem 4 of L. Fuchs in [1]. Finally, the question of whether any two high subgroups of a torsion group are isomorphic is considered, and a theorem in this direction is proved.

Preliminaries.

LEMMA 1. *Let M and N be subgroups of a primary group G such that M is neat in G and $M[p] \oplus N[p] = G[p]$. Then M is N-high in G.*

Proof. Suppose M is not N-high in G. Then there exists an N-high subgroup S of G properly containing M. Let $0 \neq s + M$ be in $(S/M)\ [p]$. By the neatness of S in G ([1], pg. 92) we may suppose that $s \in S[p]$. But this contradicts $M[p] \oplus N[p] = G[p]$, and so M is N-high in G.

As a consequence of Lemma 1, we obtain a standard

COROLLARY. ([3], pg. 24). *Let G be a primary group, and H a pure subgroup containing $G[p]$. Then $H = G$.*

Received August 10, 1960.