ON A THEOREM OF FEJÉR

FU CHENG HSIANG

1. Let

\[ T: (\tau_{nv}) \quad (n = 0, 1, 2, \cdots; \nu = 0, 1, 2, \cdots) \]

be an infinite Toeplitz matrix satisfying the conditions

(i) \[ \lim \tau_{nv} = 0 \]

for every fixed \( \nu \),

(ii) \[ \lim \sum_{\nu=0}^{\infty} \tau_{nv} = 1 \]

and

(iii) \[ \sum_{\nu=0}^{\infty} |\tau_{nv}| \leq K, \]

\( K \) being an absolute constant independent of \( n \).

Given a sequence \((S_n)\) if

\[ \lim \sum_{\nu=0}^{\infty} \tau_{nv}S_{\nu} = S, \]

then we say that the sequence \((S_n)\) or the series with partial sums \( S_n \) is summable \((T)\) to the sum \( S \).

2. Suppose that \( f(x) \) is integrable in the Lebesgue sense and periodic with period \( 2\pi \). Let

\[ f(x) \sim \frac{1}{2} a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx). \]

Let

\[ \sum_{n=1}^{\infty} n(b_n \cos nx - a_n \sin nx) = \sum B_n(x) \]

be the derived series of the Fourier series of \( f(x) \). Fixing \( x \), we write

\[ \psi_x(t) = f(x + t) - f(x - t). \]

Fejér [1] has proved the following

Received July 20, 1960.