INFINITELY DIVISIBLE PROBABILITIES ON THE HYPERBOLIC PLANE

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1. Introduction. This paper may be regarded from two points of view. First of all it presents the theory of infinitely divisible radially symmetric probability measures on the hyperbolic plane and the naturally associated limit theorems. This point of view provided the motivation for the present paper and is explained in some detail in § 2 and 3. However, just as the analagous theory in the Euclidean case may be viewed as a chapter in the theory of Fourier transforms, so may the present theory be viewed as a chapter in the theory of Legendre transforms. That is, by using the harmonic analysis described in §§ 2 and 3 one can set up a one-to-one correspondence between the radially symmetric probability measures, μ, on the hyperbolic plane and certain functions of a complex variable $\phi(z)$ in such a way that the convolution of μ_1 and μ_2 on the hyperbolic plane corresponds to the pointwise product of their "transforms" ϕ_1 and ϕ_2. Since μ is radially symmetric it is completely specified by a distribution function $F(\lambda)$ on $\lambda \geqq 0$ and the correspondence between ϕ and μ (or F) is given by

$$\phi(z) = \int_0^\infty K(z, \lambda) dF(\lambda) \tag{1.1}$$

where $K(z, \lambda)$ is a certain Legendre function given by (4.9). The convolution of μ_1 and μ_2, at least in the case where F_1 and F_2 have densities, is written down explicitly in (3.9).

This second point of view is adopted for the most part beginning in § 4 and so the majority of the paper (sections 4–10) deals with certain problems in the theory of the Legendre transform (1.1). The tools we use are those of classical analysis, but the problems treated are motivated by probability theory. The main results of the paper are contained in §§ 7 and 8. In § 10 Gaussian and stable distributions are defined within the present context. Finally in § 11 we indicate the extensions of these ideas to a wider class of Legendre transforms which includes the theory of radially symmetric probability measures on the higher dimensional hyperbolic spaces as special cases.

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