GROUPS WHICH HAVE A FAITHFUL REPRESENTATION OF DEGREE LESS THAN $(p-1/2)$

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1. Introduction. Let G be a finite group which has a faithful representation over the complex numbers of degree n. H. F. Blichfeldt has shown that if p is a prime such that $p > (2n+1)(n-1)$, then the Sylow p-group of G is an abelian normal subgroup of G [1]. The purpose of this paper is to prove the following refinement of Blichfeldt's result.

THEOREM 1. *Let p be a prime. If the finite group G has a faithful representation of degree n over the complex numbers and if $p > 2n+1$, then the Sylow p-subgroup of G is an abelian normal subgroup of G.*

Using the powerful methods of the theory of modular characters which he developed, R. Brauer was able to prove Theorem 1 in case p^2 does not divide the order of G [2]. In case G is a solvable group, N. Ito proved Theorem 1 [4]. We will use these results in our proof.

Since the group $SL(2, p)$ has a representation of degree $n = (p-1)/2$, the inequality in Theorem 1 is the best possible.

It is easily seen that the following result is equivalent to Theorem 1.

THEOREM 2. *Let A, B be n by n matrices over the complex numbers. If $A^r = I = B^s$, where every prime divisor of rs is strictly greater than $2n+1$, then either $AB = BA$ or the group generated by A and B is infinite.*

For any subset S of a group G, $C_G(S)$, $N_G(S)$, $|S|$ will mean respectively the centralizer, normalizer and number of elements in S. For any complex valued functions ζ, ξ on G we define

$$(\zeta, \xi)_G = \frac{1}{|G|} \sum_G \zeta(x)\overline{\xi(x)} ,$$

and $\|\zeta\|_G^2 = (\zeta, \zeta)_G$. Whenever it is clear from the context which group is involved, the subscript G will be omitted. $H \lhd G$ will mean that H is a normal subgroup of G. For any two subsets A, B of G, $A - B$ will denote the set of all elements in A which are not in B. If a subgroup of a group is the kernel of a representation, then we will also say that it is the kernel of the character of the given representation. All groups

Received November 25, 1960. The first author was partly supported by O. O. R. and an NSF Grant. The second author was partly supported by the Esso Research Foundation.