SELF-INTERSECTION OF A SPHERE ON A COMPLEX QUADRIC

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1. The real part S^n of a quadric V in complex, affine $(n+1)$-space is a sphere. The self-intersection of S^n *in* V is the same as the self-intersection of a "vanishing cycle," introduced by Lefschetz, and plays a certain role in [4], [5]. We will compute here this self-intersection number, using elementary tools.

Let us introduce some notations. P_{n+1} denotes the complex projective space of algebraic dimension $n+1$, hence of topological dimension

$$\dim P_{n+1} = 2n + 2 \ .$$

To each projective sub-space P_k of P_{n+1} a *positive* orientation can be given, thus it can be considered as a *cycle* p_{2k}. Then we agree that

$$\text{(1)} \qquad if\ k + l = n + 1,\ \text{then}\ (p_{2k}, p_{2l}) = 1\ \text{in}\ P_{n+1}$$

be true for the *intersection numbers* of cycles. This is the usual convention, the one in [1], for example; in [7] another convention is adopted.

Let $x_1, \cdots, x_{n+2}$ be a fixed system of projective coordinates in P_{n+1}. Then

$$\text{(2)} \qquad Q_n : x_1^2 + \cdots + x_{n+2}^2 = 0$$

is a *non-singular quadric*; $\dim Q_n = 2n$. The points of P_{n+1} whose last coordinate is non-zero form a complex affine space C_{n+1}, and

$$V = Q_n \cap C_{n+1} = [x: x \in Q_n, x_{n+2} \neq 0]$$

is a *non-singular affine quadric.* If $z \in C_{n+1}$, we denote by $z_1, \cdots, z_{n+2}$ those coordinates for which $z_{n+2} = i$ where $i^2 = -1$; thus $z_1, \cdots, z_{n+1}$ are affine coordinates in C_{n+1}. Then

$$V\colon\ z_1^2 + \cdots + z_{n+1}^2 = 1 \qquad (z \in C_{n+1})$$
$$S^n\colon\ z_1^2 + \cdots + z_{n+1}^2 = 1, z_1 \cdots, z_{n+1}\ reals$$

are the equations of an affine quadric and its real part respectively; *this real part* S^n is, of course, a sphere. We consider S^n with an arbitrarily chosen and fixed orientation as a cycle s. It is well known (see, for example, [2], p. 35, (g)) that

Received September 12, 1960.