ON THE FIELD OF RATIONAL FUNCTIONS OF ALGEBRAIC GROUPS

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0. Introduction. Let K be an algebraically closed field of characteristic 0, let k be a subfield of K and suppose that G is a (k, K) algebraic group, i.e., an algebraic group defined over k and composed of K-rational points. Let $k(G)$ denote the fields of k-rational functions on G. G_k denotes the subgroup of G composed of all k-rational points of G. If $g \in G_k$ then the regular mapping $L_g(R_g)$ of G onto G defined by $L_g x = gx$ $(R_g x = xg)$ induces an automorphism of $k(G)$ denoted by $g_l(g_r)$. Let D_k denote the Lie algebra of all k-derivations of $k(G)$ (i.e., of all derivations of $k(G)$ that are trivial on k) which commute with g_r, for every $g \in G_k$.

For any subset A of $k(G)$ let $G(A)$ denote the subgroup of G composed of all elements g such that $g_r(f) = f$, for every $f \in A$. In the sequel we shall always assume that G_k is dense in G.

The main result of this paper is the following theorem:

THEOREM 1. *Let F be a subfield of $k(G)$ containing k. Then the following three conditions are equivalent*:

(1) *F is $(G_k)_l$ — stable*

(2) *F is D_k — stable*

(3) *$F = k(G/G(F))$ and so F coincides with the field of all elements of $k(G)$ that are fixed under $G(F)_r$.*

By means of the theorem, we establish a Galois correspondence between a family of subgroups of G and the family of $(G_k)_l$-stable subalgebras of the algebra of representative functions of G.

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1. Let K be an algebraically closed field of characteristic 0, let k be a subfield of K and suppose that V, W are (k, K) — algebraic varieties. Let $k(V)$, $k(W)$ denote the fields of k-rational functions on V and W, respectively. If A is a subset of $k(V)$ then $k(A)$ denotes the fields generated by k and A.

The following result is known[1]:

(1) Let F be a rational mapping of V onto a dense subset of W and let φ be the cohomomorphism corresponding to F. Then there exists

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[1] See e.g. [2],