PREDICTION THEORY FOR MARKOFF PROCESSES

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In this paper we consider the least square prediction problem for Markoff processes with stationary transitions. The main result concerns the partial differential equation characterizing the prediction operator, and the conditions for the uniqueness of the solutions.

Introduction. Let $x(t)$ be a Markoff process with stationary transitions. It is well-known that the optimum mean square predictor of $g(x(s+t))$ given $x(\sigma)$ for $\sigma \leqq s$ is given by the conditional expectation:

$$E[g(x(t+s)) \mid x(\sigma) \leqq s] .$$

For a Markoff process this becomes

$$(1.1) \qquad E[g(x(t+s)) \mid x(s)]$$

and further, if the transitions are stationary, we need only to consider:

$$(1.2) \qquad E[g(x(t)) \mid x(0)]$$

Let $p(t, \xi \mid x)$ be the distribution function (suitably normalized) of the conditional or transition probability of transition from x to ξ in time t. Then, of course, (1.2) becomes

$$(1.3) \qquad \int g(\xi) d_\xi p(t, \xi \mid x) .$$

Now if $g(.)$ is in $C[\alpha, \beta]$, where $-\infty \leq \alpha < \beta \leq +\infty$ is the interval over which the transition probabilities are defined, we obtain a semigroup of linear operators over $C[\alpha, \beta]$ defined through (1.3). If now we know the infinitesimal generator of this semigroup, we obtain an abstract differential equation for (1.3):

$$(1.4) \qquad \frac{du(t, g)}{dt} = \Delta u(t, g)$$

where $u(t, g)$ represents (1.3) and Δ is the infinitesimal generator, provided $g(.)$ is in the domain of Δ. If we know the representation of Δ, and if in particular, it turns out to be a partial differential operator, (1.4) offers an alternate way of determing the prediction functions (1.2) provided uniqueness of the solution can be proved. In what follows, we shall be concerned primarily with situations where such a reduction is possible, and the associated conditions for uniqueness.

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