ON THE FROBENIUS RECIPROCITY THEOREM FOR LOCALLY COMPACT GROUPS

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1. Introduction. Let G be a finite group, K a subgroup, and let M and L be finite dimensional representations (over the complex numbers). Then the Frobenius reciprocity theorem (abbreviated as FRT) can be stated in two forms which are readily seen to be equivalent.

(1.) Let U^L denote the induced representation of L on G and $(M)_K$ the restriction of M to K; then there is an isomorphism

$$\psi : \mathrm{Hom}_K (L, (M)_K) \simeq \mathrm{Hom}_G (U^L, M)$$

(2.) If L and M are irreducible, then U^L contains M exactly as many times as $(M)_K$ contains L.

The first version is of course formally stronger if one specifies, as is natural, that the isomorphism ψ be functoral.

It is natural to ask how this theorem can be generalized to the context of locally compact groups. Both Mackey and Mautner have defined and discussed the notion of induced representation in this context. Each has formulated a version of the FRT for locally compact groups which generalizes the theorem as stated in form (2) above; [3], [4], [5]. These formulations, however, which use the direct integral decomposition theory, no longer embody the FRT in form (1.) above. It is the purpose of this note to show how one can obtain a version of the FRT which generalizes the theorem in form (1.). The difference between the two versions can conveniently be thought of as a distinction between a global formulation as in (1.) and a local formulation as in (2.).

Concerning our notations, we shall follow a practice throughout and use locally compact to abbreviate locally compact and separable and unitary representation to abbreviate strongly continuous unitary representation on a separable Hilbert space. The notation $\mathrm{Hom}_G (L, M)$ for representations L and M of G will naturally denote the space of bounded operators A from the space $H(L)$ on which L acts to $H(M)$ so that $AL_s = M_sA$ for all s in G. Concerning induced representations, we adopt the notation of [2].

2. Statement of the theorem. Let G be a locally compact group and K a closed subgroup. We shall assume throughout that G/K (right cosets) possesses a measure μ invariant under the operation of G by right translations on G/K. Such a measure exists if for instance G and

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