AN EXISTENCE THEOREM FOR A GOURSAT PROBLEM

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Introduction. Diaz [2] has established some global existence theorems concerning the partial differential equation $u_{xy} = f(x, y, u, u_x, u_y)$ by an analogue of the Euler-Cauchy polygon method, requiring that $u(x, y_0) = \sigma(x)$ and $u(x_0, y) = \tau(y)$ where σ and τ are of class C' on the ranges considered and $\sigma(x_0) = \tau(y_0)$, $f(x, y, z, p, q)$ is a real valued function defined for all (x,y,z,p,q) for which $x_0 \leqq x \leqq x_0 + a$; $y_0 \leqq y \leqq y_0 + b$, $-\infty < z,p,q < \infty$, is continuous and bounded over that set, and satisfies a Lipschitz condition in the last two variables.

A local existence theorem is given here in which the boundary functions have a Lipschitz condition imposed rather than being of class C', and $f(x,y,z,p,q) = g(x,y,z)p + h(x,y,z)q + j(x,y,z)$ is required to be continuous over a more restricted set than that used by Diaz resulting in the convergence of the approximating functions over a subset of the original set rather than over the entire set defined by $x_0 \leqq x \leqq x_0 + a$; $y_0 \leqq y \leqq y_0 + b$. The fact that the boundary functions need not have first derivatives defined over their entire domains of definition results in the limit function not necessarily having first partial derivatives, nor a cross derivative defined over its entire domain of definition. The notion of an ordinary derivative [7] for an interval function is used to replace the cross derivative.

This is a correction of a stronger result announced earlier by the author (Abs. 550–15, Notices, AMS No. 1958).

Most of the material in § 1 can be pieced together from the literature [3], [4], [5], [6]. The definitions and theorems needed will be stated here for convenience and in some cases proofs will be outlined as details in the method of proof are needed later in the paper and in some cases notation to be used later is established.

1. Preliminary definitions and theorems. By line interval will be meant a closed interval $a \leqq x \leqq b$. By plane interval will be meant a rectangular disk $a \leqq x \leqq b$; $c \leqq y \leqq d$. These will be denoted by $[a, b]$ and $[a, b; c, d]$. Suppose $g(x, y)$ is a function defined over $[A, B; C, D]$. We define an interval function G from g as follows. Suppose $I = [a,b;c,d] \subset [A,B;C,D]$, then $G(I) = g|_a^b|_c^d = g(a,c) - g(b,c) - g(a,d) + g(b,d)$ (with a similar notation $G(I)|_{x=a} = g(a, c) - g(a, d)$ etc., for a function of a single variable). G is an additive function. In what follows it is assumed that all points and intervals considered are in $[A, B; C, D]$. It is easily seen that adding to g functions of the single variable x or y does

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