IDEMPOTENT MEASURES ON SEMIGROUPS

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Introduction. Some interest has been shown in the problem of determining idempotent measures on topological groups and, more recently, on semigroups. Wendel [10] seems to have opened the subject in 1954 with the pleasing result that the positive idempotent measures on a compact group were precisely the (normalized) Haar measures of compact subgroups. In 1959 Rudin [6] showed that the same result held for locally compact abelian groups, and in 1960 Cohen [1] determined all idempotents (real and complex) on such groups. Glicksberg [2] (1959) showed that, on a compact abelian semigroup, to be the Haar measure of a compact subgroup was equivalent to being a positive idempotent.

In the present paper, the problem is considered for locally compact semigroups.

The problem is solved for the general locally compact group by Theorem 4.1 of §A, in which it is also shown that the support of an idempotent measure on certain types of locally compact semigroups (which include compact semigroups) is a compact kernel (definition in §B). In §B we describe the structure of compact kernels, giving results obtaing by Wallace [9] as a preliminary to describing the idempotent measures on them in §C. The relationships between invariant and idempotent measures are given in Theorems C4.1 and C5.1. Section C closes with a discussion of primitive idempotents which we see in §D are important in the structure of the semigroup of measures on a compact semigroup.

There is some slight overlap between the results given here and those published recently by Collins (Proc. Amer. Math. Soc. 13 (1962), 442–446, and Duke Math. J., 28 (1961), 387–392).

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A. Idempotent measures on locally compact semigroups.

1. The set of bounded, Borel measures on a locally compact semigroup S forms a Banach algebra when it is given the norm it acquires as the dual of $\mathfrak{K}(S)$ (the space of complex-valued continuous functions of compact support on S with the uniform norm) and when multiplication is defined by convolution: $\mu * \nu(\varphi) \equiv \int_S \int_S \varphi(xy)\, d\mu(x)\, d\nu(y)$ for $\varphi \in \mathfrak{K}$. The measure μ is said to be concentrated on a set E if, whenever the support of φ, (S_φ), is disjoint from E, $\mu(\varphi) = 0$. The support (S_μ) of μ is the smallest closed set on which it is concentrated.

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