MARKOV PROCESSES WITH STATIONARY MEASURE

S. R. FOGUEL

In [1] we studied Markov processes with a finite positive stationary measure. Here the process is assumed to have a positive stationary measure which might be infinite. Most of the results proved in [1] remain true also in this case. Some proofs that remain valid in this case will be replaced here by simpler proofs.

The main problem studied here, and in [1], is the behaviour at ∞ of $\mu(x_n \in A \cap x_0 \in B)$ where μ is the stationary measure and x_n is the Markov process.

In addition we study the quantities

$$\mu(x_n \in A \text{ for some } n \cap x_0 \in B)\ , \quad \mu((x_n \in A \text{ infinitely often}) \cap x_0 \in B)\ .$$

For Markov chains the results given here are well known even without the assumption of the existence of a stationary measure.

DEFINITIONS AND NOTATION. The notation here will be the same as in [1]. Let (Ω, Σ, μ) be a measure space where $\mu \geqq 0$ but is not necessarily finite.

Let $x_n(\omega)$ be a sequence of measurable real functions defined on Ω.

Let the measure $\mu(x_0^{-1}(\quad))$, on the real line, be σ finite.

ASSUMPTION 1. *The process is stationary*:

$$\mu(x_{n+k} \in A \cap x_{m+k} \in B) = \mu(x_n \in A \cap x_m \in B)\ .$$

ASSUMPTION 2. *If $i < j < k$ let A be a Borel set on the line such that $\mu(x_k \in A) < \infty$ then*:

The conditional probability that $x_k \in A$, given x_j and x_i, is equal to the conditional probability that $x_k \in A$ given x_j.

$L_2 = L_2(\Omega, \Sigma, \mu)$ will be the space of real square integrable function. Let B_n be the subspace of L_2 generated by functions of the form

$$I(x_n^{-1}(A)) \text{ where } \mu(x_n^{-1}(A)) < \infty\ .$$

By $I(\sigma)$ we denote the characteristic function of σ.

Let E_n be the self adjoint projection on B_n.

It was shown in [1] that Assumption 2 implies

1. $$E_i E_j E_k = E_i E_k \qquad i < j < k\ .$$

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