A CONTINUITY PROPERTY FOR VECTOR VALUED MEASURABLE FUNCTIONS

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1. Introduction. The principal purpose of this paper is to characterize certain types of Banach spaces of vector valued and integrable functions and their conjugate or adjoint spaces, and to apply these characterizations to obtain an effective way of determining the adjoint space of the Banach space $R[a, b]$, where $[a, b]$ is a real number interval, $R[a, b]$ is the space of Riemann integrable functions on $[a, b]$, and for $f \in R[a, b]$, $\|f\| = \sup [|f(x)|; a \leqq x \leqq b]$. We use the term effective in the sense that we want our determination of the adjoint space of $R[a, b]$ to enable us to probe deeper into its analytic structure in order to, say for example, obtain a weak convergence and compactness theory.

We are interested in spaces of vector valued functions which arise in the following manner. Let X be a set, B a Banach space, and f a a vector valued function on X to B. We call f partitionable if for each $\varepsilon > 0$, there exists a finite partition $[E_i; i \leqq n]$ of X such that $\max [0(f, E_i); i \leqq n] < \varepsilon$, where $0(f, E_i) = \sup [\|f(x) - f(y)\|; x, y \in E_i]$. Hence, if S is an algebra of subsets of X and g is a bounded and finitely additive set function on S, then the collection of bounded, B-valued, partitionable, and g-measurable functions on X (defined in § 2) form a Banach space which we denote by $m(X, S, g, B)$ (the norm of the elements f of $m(X, S, g, B)$ is $\sup [\|f(x)\|; x \in X]$). The space $m(X, S, g, B)$ is the type of space we characterize and we do it by isomorphically and isometrically embedding $m(X, S, g, B)$ onto the space $C(X, T, B)$ of B-valued functions defined on X which are continuous (Definition 2.4) with respect to an algebra T of subsets of X. The set T is essentially the g-outer measure completion of S. Then we characterize the adjoint space $C^*(X, T, B)$ of the space $C(X, T, B)$ by embedding $C^*(X, T, B)$ isomorphically and isometrically onto a space of bounded B^*-valued finitely additive set functions on T (defined in § 3) and, consequently, we can easily find the adjoint space $m^*(X, S, g, B)$. In the case where B is the real number system (henceforth we shall denote by $\mathscr{R}$ the real number system) we simply denote $m(X, S, g, B)$ by $m(X, S, g)$. The space $R[a, b]$ turns out to be a realization of this type of space and it is by this method that we characterize $R^*[a, b]$.

The text of this paper consists of three sections. In § 2 we present several definitions and introduce some notation and terminology. In § 3 our principal results (Theorems 3.2, 3.3, and 3.4) give a characterization of the spaces $m(X, S, g, B)$, $C^*(X, S, B)$, and $m^*(X, S, g, B)$ respectively,

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