DECOMPOSITION AND HOMOGENEITY OF CONTINUA ON A 2-MANIFOLD

H. C. WISER

1. Introduction. Many partial results have been obtained in attempting to characterize homogeneous plane continua; a history of this problem can be found in [4]. The question arises; which of these results hold for homogeneous proper subcontinua of a 2-manifold, and indeed do there exist such continua which cannot be embedded in the plane? The main purpose of this paper is to extend some results for plane homogeneous continua to corresponding results for continua on a 2-manifold, with a long range aim of investigating the embedding problem.

Let X be a nondegenerate homogeneous plane continuum. F. B. Jones [10] has shown that X is a simple closed curve if it is aposyndetic or if it contains a noncutpoint, H. J. Cohen [7] has shown that X is a simple closed curve if it either contains a simple closed curve or is arcwise connected, and R. H. Bing [3] has shown that X is a simple closed curve if it contains an arc. In § 4 the above results of Cohen's and Jones' are generalized to homogeneous continua on 2-manifolds. Section 3 contains results on collections of continua which arise rather naturally in considering the generalizations of Cohen's work.

Jones [12] has shown that if X is decomposable and is not a simple closed curve, at least it becomes one under a natural aposyndetic decomposition. In § 5 this result is extended to homogeneous continua on a 2-manifold as well as to homogeneous continua with a multicoherence restriction.

In extending plane results to results on arbitrary 2-manifolds, we will use as a generalization of the Jordan curve theorem the fact that for any 2-manifold M there exists a positive integer k such that M is separated by the sum of any k disjoint simple closed curves on M.

2. Definitions. Only separable metric spaces will be considered here. A connected compact metric space is called a *continuum*. A 2-*manifold* is a continuum such that each of its points lies in an open set topologically equivalent to Euclidean 2-space. A 2-*manifold with boundary* is a continuum such that each of its points lies in an open set whose closure is topologically equivalent to a closed 2-cell.

A point set X is said to be *n-homogeneous* if for any n points $x_1, x_2, \cdots, x_n$ of X and any n points $y_1, y_2, \cdots, y_n$ of X there is a home-

Received July 5, 1961. This paper is part of a thesis submitted to the faculty of the University of Utah in partial fulfillment of the requirements for the Ph. D. degree, June, 1961. The author is indebted to Professor C. E. Burgess for his encouragement and advice.