A NOTE ON THE PRIMES IN A BANACH ALGEBRA OF MEASURES

JAMES WELLS

1. Introduction. Let V denote the family of all finite complex-valued and conuntably additive set functions on the Borel subsets of $R_+ = [0\ \infty)$ (hereafter called measures); $L^1(R_+)$ the set of all complex-valued functions on R_+ which are integrable in the sense of Lebesgue, identifying functions which are 0 almost everywhere; and A the elements in V which are absolutely continuous with respect to Lebesgue measure. For each $\mu \in V$ there exists an $f \in L^1(R_+)$ such that

$$\mu(E) = \int_E f(x)dx \tag{1.1}$$

for each Borel subset E of R_+. And, conversely, if $f \in L^1(R_+)$ the set function μ defined by (1.1) is a measure.

We introduce a norm into V by the formula

$$\| \mu \| = \sup \Sigma \,| \mu(E_i) | \qquad (\mu \in V)\ , \tag{1.2}$$

the supremum being taken over all finite partitions of R_+ into pairwise disjoint Borel sets E_i. It is well known ([6], p. 142 or [7]) that V becomes a commutative Banach algebra under the convolution operation

$$\nu(E) = \int_0^\infty \mu(E - x)d\lambda(x) \qquad (\mu, \lambda \in V)\ , \tag{1.3}$$

where E is any Borel subset of R_+; in symbols

$$\nu = \mu * \lambda\ . \tag{1.4}$$

The Laplace-Stieltjes transform of $\mu \in V$ will be denoted by $\hat{\mu}$:

$$\hat{\mu}(z) = \int_0^\infty e^{-zx}d\mu(x) \qquad (Re(z) \geqq 0)\ . \tag{1.5}$$

The relation (1.4) is equivalent to

$$\hat{\nu}(z) = \hat{\mu}(z)\hat{\lambda}(z) \qquad (Re(z) \geqq 0)\ . \tag{1.6}$$

The *identity* in V is the measure u such that $u(E) = 1$ if $0 \in E$ and 0 otherwise. A measure μ is *invertible* provided there exists a measure μ^{-1} such that $\mu * \mu^{-1} = u$; and the measure λ is a *divisor* of the measure μ, in symbols $\lambda \mid \mu$, provided there exists a measure ν such that $\mu = \lambda * \nu$. It follows from basic properties of the Laplace-Stieltjes

Received December 4, 1961.