MEROMORPHIC FUNCTIONS AND CONFORMAL METRICS ON RIEMANN SURFACES

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1. The starting point of the present paper is the classical theory of meromorphic functions in the plane or the disk. We shall generalize fundamentals of this theory to open Riemann surfaces W_s that carry a specified conformal metric (Nos. 3, 11). The motivation is that meromorphic functions are defined by a local property and it is natural to consider them on the corresponding locally defined carrier, a 2-manifold with conformal structure.

The method we shall use largely parallels that of F. Nevanlinna [10] and L. Ahlfors [1]. We have, however, made an effort to write the presentation self-contained. The classical theory will be included as a special case.

We note in reference to earlier work generalizations given in various directions by L. Ahlfors [2], S. Chern [4, 5], G. af Hällström [6], K. Kunugui [8], L. Myrberg [9], L. Sario [14, 15], J. Tamura [19], Y. Tumura [22], and M. Tsuji [21].

2. Our principal result will be the integrated (Nevanlinna) form of the second main theorem on W_s (No. 17). No generalization of this theorem to Riemann surfaces of arbitrary genus has, to our knowledge, been given thus far. As a corollary the following extension of Picard's theorem will be established: Let P be the number of Picard values of a meromorphic function w on a Riemann surface W_p with the capacity metric (No. 21). Form the characteristic function $T(h)$ of w on the region W_h bounded by the level line $p_\beta = h$ of the capacity function p_β. Denote by $E(h)$ the integrated Euler characteristic of W_h and set

$$\eta = \overline{\lim} \frac{E(h)}{T(h)} .$$

Then

$$P \leqq 2 + \eta .$$

This bound is sharp (No. 27). Analogous extensions will be given to other classical consequences of the second main theorem (Nos. 31–36).

A generalization to arbitrary Riemann surfaces of the nonintegrated form of the second main theorem is given in [18].

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