SIMPLE MALCEV ALGEBRAS OVER FIELDS OF CHARACTERISTIC ZERO

ARTHUR A. SAGLE

1. Introduction. Malcev algebras are a natural generalization of Lie algebras suggested by introducing the commutator of two elements as a new multiplicative operation in an alternative algebra [3]. The defining identities obtained in this way for a Malcev algebra A are

$$xy = -yx \tag{1.1}$$

$$xy \cdot xz = (xy \cdot z)x + (yz \cdot x)x + (zx \cdot x)y \tag{1.2}$$

for all $x, y, z \in A$. Since Albert [1] has shown that every simple alternative ring which contains an idempotent not its unity quantity is either associative or the split Cayley-Dickson algebra C, it is natural to see if a simple Malcev algebra can be obtained from C. In [3] a seven dimensional simple non-Lie Malcev algebra A^* is obtained from C and is discussed in detail. In this paper we shall prove the following

THEOREM. *Let A be a finite dimensional simple non-Lie Malcev algebra over an algebraically closed field of characteristic zero. Furthermore assume A contains an element u such that the right multiplication by u, R_u, is not a nilpotent linear transformation. Then A is isomorphic to A^*.*

The necessary identities and notation from [3] for any algebra A are repeated here for convenience:

$$\text{Commutator,}\quad (x, y) = [x, y] = xy - yx \tag{1.3}$$

$$\text{Associator,}\quad (x, y, z) = xy \cdot z - x \cdot yz \tag{1.4}$$

$$\text{Jacobian,}\quad J(x, y, z) = xy \cdot z + yz \cdot x + zx \cdot y \tag{1.5}$$

for $x, y, z \in A$. If $h(x_1, \cdots, x_n)$ is a function of n indeterminates such that for any n subsets B_i of A and $b_i \in B_i$, the elements $h(b_1, \cdots, b_n)$ are in A, then $h(B_1, \cdots, B_n)$ will denote the linear subspace of A spanned by all of the elements $h(b_1, \cdots, b_n)$.

For a Malcev algebra A of characteristic not 2 or 3, we shall use the following identities and theorems from [3]:

$$J(x, y, xz) = J(x, y, z)x \tag{1.6}$$

Received September 2, 1961. The author would like to thank Professor L. J. Paige for his assistance in the preparation of the manuscript. This research was sponsored in part by the National Science Foundation under NSF Grant G-9504.