EVALUATION OF AN E-FUNCTION WHEN THREE OF ITS UPPER PARAMETERS DIFFER BY INTEGRAL VALUES

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1. Introduction. If $p \geqq q+1$, [1, p. 353]

$$E(p; \alpha_r : q; \rho_s : z) = \sum_{r=1}^{p} z^{\alpha_r} \sum_{n=0}^{\infty} \frac{\Gamma(\alpha_r + n) \prod_{t=1}^{p}{}' \Gamma(\alpha_t - \alpha_r - n)}{n! \prod_{s=1}^{q} \Gamma(\rho_s - \alpha_r - n)} (-z)^n , \tag{1}$$

where, if $p = q+1$, $|z| < 1$. The dash in the product sign indicates that the factor for which $t = r$ is omitted.

Now, if two or more of the α's are equal or differ by integral values, some of the series on the right cease to exist. For instance, if $\alpha_1 = \alpha + l$, $\alpha_2 = \alpha$, where l is zero or a positive integer, it has been shown [2, p. 30] that the first two series can be replaced by the expression

$$\begin{aligned} &(-1)^l z^{\alpha+l} \sum_{n=0}^{\infty} \frac{\Gamma(\alpha + l + n) \prod_{t=3}^{p} \Gamma(\alpha_t - \alpha - l - n)}{n!(l+n)! \prod_{s=1}^{q} \Gamma(\rho_s - \alpha - l - n)} \Delta_n z^n \\ &+ z^{\alpha} \sum_{n=0}^{l-1} \frac{\Gamma(\alpha + n)(l-n-1)! \prod_{t=3}^{p} \Gamma(\alpha_t - \alpha - n)}{n! \prod_{s=1}^{q} \Gamma(\rho_s - \alpha - n)} (-z)^n , \end{aligned} \tag{2}$$

where

$$\begin{aligned} \Delta_n = {} & \psi(l+n) + \psi(n) - \psi(\alpha + l + n - 1) - \log z \\ & + \sum_{t=3}^{p} \psi(\alpha_t - \alpha - l - n - 1) - \sum_{s=1}^{q} \psi(\rho_s - \alpha - l - n - 1) . \end{aligned}$$

Here

$$\psi(z) = \frac{d}{dz} \log \Gamma(z+1) , \tag{3}$$

so that

$$\frac{d}{dz} \Gamma(z+1) = \Gamma(z+1) \psi(z) . \tag{4}$$

It will now be shown that, in the case in which

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