THE BEHAVIOR OF SOLUTIONS OF ORDINARY, SELF-ADJOINT DIFFERENTIAL EQUATIONS OF ARBITRARY EVEN ORDER

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Introduction. It is the purpose of this paper to establish some properties of the zeros of solutions of ordinary, self-adjoint differential equations of arbitrary even order of the form

$$[r(x)y^{(n)}]^{(n)} + (-1)^{n+1}p(x)y = 0 \tag{1}$$

where $r(x) > 0$, $p(x) > 0$, and both coefficients are continuous on $[a, \infty)$. Of particular concern is the existence of a nontrivial solution of (1) which satisfies one of the following sets of two-point boundary conditions

$$y(a) = y'(a) = \cdots = y^{(n-1)}(a) = 0 = y(b) = y'(b) = \cdots = y^{(n-1)}(b) \tag{2}$$

$$y(a) = y'(a) = \cdots = y^{(n-1)}(a) = 0 = y_1(b) = y_1'(b) = \cdots = y_1^{(n-1)}(b) \tag{3}$$

where $y_1(x) \equiv r(x)y^{(n)}(x)$, a notation which will be continued throughout the discussion, and $b > a$.

Recently the special fourth-order case ($n = 2$) has been investigated extensively by W. Leighton and Z. Nehari [10], by H. M. and R. L. Sternberg [13], by H. C. Howard [8], and by J. H. Barrett [2, 3, 4]. In the present paper some of the methods of Barrett [2, 4] are extended to the general case; and, in so doing, some of the arguments used for $n = 2$ are simplified.

W. T. Reid has recently announced [12] a general discussion including the above types of zeros of solutions of quasi-differential equations of even order of which (1) is a special case. Reid discusses related eigenvalue inequalities and his methods are variational in nature and assume some basic results of the spectral theory for boundary problems that have been established earlier in the study of the calculus of variations.

This discussion, which generalizes Barrett's methods, has the advantage that only fairly well-known properties of matrices and differential equations are used. Furthermore, and most important, a considerably stronger criterion for the existence of a non-trivial solution satisfying (2) (see Theorem 4.3) and of one satisfying (3) (see Corollary 5.1) is established by utilizing the simple form of (1). Then two comparison theorems, established by an application of Reid's variational results [12], extend these stronger results to the general self-adjoint

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