ANNIHILATORS IN THE SECOND CONJUGATE ALGEBRA OF A GROUP ALGEBRA

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1. Introduction. Let $\mathfrak{G}$ denote an infinite locally compact abelian group, and let $L(\mathfrak{G})$ be its group algebra. The second conjugate space $L^{**}(\mathfrak{G})$ of the group algebra can also be considered as an algebra by the use of Arens multiplication [1] [2]. Civin and Yood [3, p. 857] have shown that $L^{**}(\mathfrak{G})$ is an algebra which is not commutative and has a nonzero radical $\mathfrak{R}^{**}$. They have also shown [3, p. 856] that if $\mathfrak{G}$ is not discrete, then the algebra $L^{**}(\mathfrak{G})$ has a nonzero right annihilator.

The object of the present note is the study of the nature of the left and right annihilators of the maximal modular left ideals in $L^{**}(\mathfrak{G})$. It is shown that such annihilators are either nilpotent two-sided or right ideals, respectively, or else the maximal modular left ideal in question must have the form $\{F \in L^{**}(\mathfrak{G}) \mid F(\mu) = 0\}$ where μ is some multiplicative linear functional on $L(\mathfrak{G})$. If $\mathfrak{G}$ is compact it is seen that all maximal modular left ideals of the latter form have a nonzero left annihilator and a right annihilator which properly contains the right annihilator of $L^{**}(\mathfrak{G})$.

It should be noted that the choice of the maximal modular left ideals as the subject of investigation is not simply for definiteness. At the present stage of available information concerning $L^{**}(\mathfrak{G})$, the maximal modular left ideals are more tractable than the corresponding right ideals.

2. Notation. Throughout the note we shall use the notation introduced above as well as other notation introduced by Civin and Yood [3]. In particular $\mathfrak{R}^{**}$ will denote the radical of $L^{**}(\mathfrak{G})$ and $\mathfrak{Y}$ will denote the closed subspace of $L^{*}(\mathfrak{G})$ generated by the multiplicative linear functionals on $L(\mathfrak{G})$. We shall write $\mathfrak{L}(I)(\mathfrak{R}(I))$ for the left (right) annihilators in the algebra $L^{**}(\mathfrak{G})$ of the subset I of $L^{**}(\mathfrak{G})$. We also use the notation $I^{\perp}(I^{\top})$ for the linear space annihilator in $B^{*}(B)$ of the linear manifold I in the Banach space B (the conjugate space B^{*}). Throughout π will be used for the natural embedding of a Banach space B into its second conjugate space B^{**}. It should be recalled [1] that when B is a Banach algebra, π is an algebra homomorphism, and if B is commutative then [3, p. 855] πB is in the center of B^{**}.

3. Left annihilators. Throughout this section we let $\mathfrak{M}$ denote a maximal modular left ideal in $L^{**}(\mathfrak{G})$ for which $\mathfrak{L}(\mathfrak{M}) \neq (0)$.

LEMMA 3.1. *$\mathfrak{M}$ and $\mathfrak{L}(\mathfrak{M})$ are 2-sided ideals in $L^{**}(\mathfrak{G})$ and*

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