ON THE ANALYTIC SPECTRUM OF ARENS

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The object of this note is to show that the "analytic spectrum" of $a = (a_1, \cdots, a_n)$ defined by Arens [1] is the complement of the largest open set X in C^n, on which locally bounded functions, $u_1(s), \cdots, u_n(s)$ can be found, which satisfy identically the equation

$$\Sigma(a_i - s_i)u_i(s) = 1 .$$

We shall have to prove that the existence of bounded functions satisfying this relation in the neighbourhood of the point t implies the existence of analytic functions, which satisfy this equation on a neighbourhood of t that may be smaller than the original one. The main tool in our proof will be statement B, p. 101, of [4].

1. Arens calls a "topological algebra" an algebra, $\mathscr{A}$, with a locally convex structure making the product a continuous mapping of $\mathscr{A} \times \mathscr{A}$ into $\mathscr{A}$, and having further the property that the closed convex span of a compact set is compact. The first thing we shall show is that $\mathscr{A}$ is made into an "algèbre à bornés complète" according to the terminology of [3], when we define as bounded for the structure of $\mathscr{A}$ precisely those sets which are bounded for the topology. It will clearly be sufficient to show that absolutely convergent series converge, that is, that $\Sigma z_i b_i$ converges when b_i is a bounded sequence of $\mathscr{A}$ while z_i is a sequence of positive reals such that $\Sigma z_i < \infty$.

The sequence of partial sums of the series is obviously a Cauchy sequence. To prove it converges, it is sufficient to show that it is a sequence of elements of a compact set. But we can always find sequences y_i, z'_i, which are positive, real, and such that $y_i \to 0$, $\Sigma z'_i < \infty$, $z_i = y_i z'_i$ (when $\Sigma z_i < \infty$). We let $b'_i = y_i b_i$.

The sequence b'_i converges to zero. The points b'_i are the elements of a set whose closure is compact, whose closed convex span is therefore compact. Let B' be that closed convex span.

Let $M = \Sigma z'_i$. The partial sums of the series $\Sigma z_i b_i = \Sigma z'_i b'_i$ belong to MB'. This proves the series converges. Hence $\mathscr{A}$ is a b-algebra.

2. We shall simplify the terminology by calling henceforth the "espaces à bornés complets" and the "algèbres à bornés complètes" of [4]: "b-spaces" and "b-algebras". The results we shall use being generally true for b-algebras, it is interesting to have conditions on an algebra $\mathscr{A}$ on which a locally convex structure is besides defined, which ensure that $\mathscr{A}$ is a b-algebra, when we define as bounded the

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