ASYMPTOTIC ESTIMATES FOR LIMIT POINT PROBLEMS

C. A. SWANSON

Introduction. The variation of characteristic values and functions of the differential operator L defined by

$$Lx = \frac{1}{k(s)}\left\{-\frac{d}{ds}\left[p(s)\frac{dx}{ds}\right] + q(s)x\right\}$$

will be studied when the domain of L varies because of a change of boundary conditions. The *basic* interval is an open interval $\omega_- < s < \omega_+$ on which k is positive and piecewise continuous, p is positive and differentiable, and q is real-valued and piecewise continuous. For a closed subinterval $[a, b]$ of the basic interval, our purpose is to obtain estimates for the characteristic values μ_{ab} and characteristic functions y_{ab} of regular Sturm-Liouville problems on $[a, b]$ when a, b are near ω_-, ω_+. Such results have been obtained by the author [6] in the case that both ω_- and ω_+ are limit circle singularities in H. Weyl's classification [2, p. 225]. Here the analogous results will be derived in the limit point case and the mixed case (one singularity of each type). To avoid repetition of the preliminary material in [6], we shall usually adhere to the notation and numbering system of [6] without further comment.

6. Basic problems in the limit point and mixed cases. As in § 2, the limits of μ_{ab} as $a \to \omega_-, b \to \omega_+$ are supposed to exist, and accordingly we shall assume that characteristic values λ of suitable singular Sturm-Liouville problems for L on (ω_-, ω_+) exist. These singular problems are described as follows when both ω_-, ω_+ are limit point singularities [4].

Let L_0 be the differential operator $L - l_0$, $Im\, l_0 \neq 0$. According to a theorem of Weyl [4, p. 45] there exist linearly independent solutions φ_-, φ_+ of $L_0\varphi = 0$ such that

$$\varphi_+ \in \mathfrak{F}_{\omega\omega_+}\,, \qquad \varphi_- \in \mathfrak{F}_{\omega_-\omega}\,, \qquad [\varphi_+\bar{\varphi}_-](s) = 1 \tag{6.1}$$

for any ω satisfying $\omega_- < \omega < \omega_+$. These solutions are uniquely determined from the normalization condition $[\varphi_+\varphi_+](s_0) = i$ at some point s_0, to remain fixed in the sequel. (Compare (6.1) with the choice (2.1) of φ_-, φ_+ in the limit circle case.) Let $\mathfrak{D}^0$ be the set of all x in the basic Hilbert space $\mathfrak{H}$ (described in § 1) which have the following properties: (a) x is differentiable on (ω_-, ω_+) and x' is absolutely

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