ON THE NÖRLUND SUMMABILITY OF FOURIER SERIES

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1. Let $f(x)$ be a function integrable $-L$ over the interval $(-\pi, \pi)$ and periodic with period 2π, outside this interval.

Let

$$\phi(t) = \tfrac{1}{2}\{f(x+t) + f(x-t) - 2s(x)\}\ , \tag{1.1}$$

and

$$\tfrac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \operatorname{Sin} nx) \tag{1.2}$$

be the Fourier Series of the function $\phi(t)$.

Nörlund Summability of Fourier Series (1.2) has been considered by Woronoi [6] and later on by Nörlund [4]. These results have been extended by Hille and Tamarkin [2], [3], and later on by Astrachan [1]. Recently, extending a result due to Hille and Tamarkin [3], Varshney [5] has proved the proved the following:

THEOREM. V. *If the sequence* $\{p_n\}$ *satisfies the following conditions*:

$$\frac{n\,|\,pn\,|}{\log n} < c\,|\,P_n\,|\ , \tag{1.3}$$

$$\sum_{k=0}^{n}\frac{k\,|\,p_k - p_{k-1}\,|}{\log(k+1)} < c\,|\,P_n\,| \tag{1.4}$$

and

$$\sum_{k=0}^{n}\frac{P_k}{k\log(k+1)} < c\,|\,P_n\,| \tag{1.5}$$

and also if

$$\bar{\Phi}_1(t) = \int_0^t |\,\phi(u)\,|\,du = 0\left(t/\log\frac{1}{t}\right) \tag{1.6}$$

then the Fourier Series (1.2) *associated with the function* $\phi(t)$ *is summable by Nörlund means i.e. summable* (N, p_n) *to the sum zero at the point* $t = x$.

The object here is to prove the following:

THEOREM. *If the sequence* $\{p_n\}$ *satisfies the following conditions*

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