GENERAL GROUP EXTENSIONS

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Introduction. The aim of this paper is to show how some of the methods useful in studying normal extensions of groups can be used in a problem of more general extensions. The present approach (which might be compared with that of Szep [5]) is made possible because we consider classes of extensions which are still relatively restricted.

If G is an arbitrary subgroup of a group H then the set of all right cosets of G in H forms a mixed group under a naturally defined operation (Loewy [3]). In particular, when G is normal in H then the corresponding mixed group is the ordinary quotient group H/G. This paper is concerned with examining properties of the class of those extensions H of a given group G for which the corresponding mixed group is isomorphic to a given mixed group Γ. As an example of the results, Theorems 2.2 and 2.3 represent analogues of the corresponding theorems of Schreier on factor sets for normal extensions.

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Mixed groups.

1.1 DEFINITION. A *mixed group* is a set Γ on which a product $\alpha\beta \in \Gamma$ is defined for certain pairs $\alpha, \beta \in \Gamma$ such that

(i) a nonempty subset Δ of Γ forms a group under the given product and is called the *nucleus* of Γ;

(ii) for all $\beta \in \Gamma, \alpha\beta$ is defined if and only if $\alpha \in \Delta$; furthermore, $\alpha\beta = \beta$ if and only if $\alpha = 1$, the identity of Δ;

(iii) if $\alpha, \beta \in \Delta$ and $\gamma \in \Gamma$ then $\alpha(\beta\gamma) = (\alpha\beta)\gamma$. (See Loewy [3] and Bruck [2; page 35]. The general properties of mixed groups are derived in Baer [1].)

In particular, if H is a group with a subgroup G then the set of all right cosets of G in H forms a mixed group when the product of two elements is defined by $(Gx)(Gy) = Gxy$ whenever $x \in N(G; H)$, the normaliser of G in H. In this case we denote the mixed group by H/G and note that its nucleus is the quotient group $N(G; H)/G$ (See Baer [1]).

1.2 DEFINITION. Two mixed groups Γ and Γ' with nuclei Δ and Δ' respectively are *isomorphic* under a mapping τ if τ is a one-to-one

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