STRONGLY REGULAR GRAPHS, PARTIAL GEOMETRIES AND PARTIALLY BALANCED DESIGNS

R. C. BOSE

0. Summary. This paper introduces the concept of a *partial geometry*, which serves to unify and generalize certain theorems on embedding of nets, and uniqueness of association schemes of partially balanced designs, by Bruck, Connor, Shrikhande and others. Certain lemmas and theorems are direct generalizations of those proved by Bruck [5], for the case of nets, which are a special class of partial geometries.

1. Introduction. We use graph theoretic methods for the study of association schemes of partially balanced incomplete block (PBIB) designs. For this purpose it is convenient to switch from graph theoretic language to the language of designs and vice versa as necessary.

As we shall be concerned with finite graphs only, we shall use the word graph in the sense of finite graphs.

A graph G with v vertices is said to be regular if each vertex is joined to n_1 other vertices, and unjoined to n_2 other vertices. Clearly

$$(1.1) \qquad v - 1 = n_1 + n_2 .$$

If further any two joined vertices of G, are both joined to exactly p_{11}^1 other vertices, and any two unjoined vertices are both joined to exactly p_{11}^2 other vertices, then the graph G is defined to be *strongly regular*, with parameters

$$(1.2) \qquad n_1, n_2, p_{11}^1, p_{11}^2 .$$

The concept of strongly regular graphs is isomorphic with the concept of association schemes of PBIB designs (with two associate classes), which was first introduced by Bose and Shimamoto [4]. Such a scheme they defined as a scheme of relations between v treatements such that

(i) any two objects are either first associates or second associates