DIVISORIAL VARIETIES

MARIO BORELLI

Introduction. The purpose of the present work is to introduce a new type of algebraic varieties, called Divisorial varieties. The name comes from the fact that the topology of these varieties is determined by their positive divisors. See §3 for a more detailed discussion of the above statement.

In the first two sections we lay the groundwork for our study. The result obtained in Proposition 2.2 is new, and constitutes a natural generalization of a well known result of Serre. (See [3], page 235, and Lemma 2, page 98 of [5]).

Section 3 is devoted to the study of the categorical properties of divisorial varieties. We prove that locally closed subvarieties of divisorial varieties are divisorial, and that products and direct sums of divisorial varieties are divisorial. Furthermore we give a characterization of divisorial varieties which shows how such varieties are a natural generalization of the notion of projective varieties.

We show in §4 that all quasi-projective, and all nonsingular varieties are divisorial. A procedure is also given for constructing a large class of divisorial varieties which are neither quasi-projective nor nonsingular, both reducible and irreducible ones.

In §5 we study the additive group of equivalence classes (under linear equivalence) of locally linearly equivalent to zero divisors of a divisorial variety. We show that such group is generated by the semigroup of those classes which contain some positive members. As a matter of fact the statement of Corollary 5.1 is more general than the one above, but we omit the details here for brevity's sake. The results of §5 are a generalization of the operation of "adding hypersurface sections," well known to the Italian geometers for projective varieties.

Finally, in §6, we give one instance of a theorem which is known to be true for either quasi-projective or irreducible and nonsingular varieties, and show that it holds for divisorial varieties. The theorem considered, which we refer to as the polynomial theorem of Snapper, is Theorem 9.1 of [6], generalized by Cartier (See [1]) to either quasi-projective or irreducible and nonsingular varieties.

We believe that the notion of divisorial varieties represents a natural extension of the notion of quasi-projective varieties.

Our notation and terminology are essentially those of [3]. The word sheaf always means, unless other-wise specified, algebraic coherent

Received July 25, 1962.