ON THE COMPACTNESS OF INTEGRAL CLASSES

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1. Introduction. In a previous paper, [8], integral currents were used to develop a concept for non-oriented domains of integration in Euclidean n-space. This concept has been designed to be useful in the calculus of variations and this, therefore, demands that the domains of integration satisfy certain "smoothness" and "compactness" conditions. It was shown in [8] that these non-oriented domains, which are called integral classes, do possess the desired smoothness property and it was also shown that the integral classes possess the following compactness property: every N-bounded sequence of k-dimensional integral classes has a subsequence which converges to some flat class. In the case that $k = 0, 1, n-1$, or n, it was shown that the limiting flat class is, in fact, a rectifiable class, and therefore, a desirable compactness property is obtained.

The main purpose of this paper is to extend this compactness property to integral classes of arbitrary dimension under the assumption that certain "irregular" sets have zero measure (3.1). This is accomplished with the help of a theorem concerned with the behavior of the density of a measure associated with a minimizing sequence (2.8), and by relying heavily on the tangential properties of rectifiable sets. In the case of the Plateau Problem, two theorems concerning densities are proved (2.3, 2.4) which are analogous to results obtained in [6] and [3; 9.13].

Most of this work depends upon the paper [8], and therefore, the terminology and notation of [8] is readopted here without change. It will be assumed throughout that $1 < k < n-1$.

2. Densities. In this section, the Plateau Problem is formulated in terms of integral classes and two theorems are proved which are analogous to results obtained in [3; 9.13] and [6]. Theorem 2.8 asserts that a portion of the irregular set, A_3, which appears in (3) below, has zero measure. A similar result, which states that $D_k^*(\mu, R^n, x) < \infty$ μ-almost everywhere and therefore that $\mu(A_3) = 0$, is still lacking.

2.1. DEFINITION. If μ is a measure over R^n, $A \subset R^n$, $\alpha(k)$ the volume of the unit k-ball, and $x \in R^n$, then

$$D_k(\mu, A, x) = \lim_{r \to 0} \alpha(k)^{-1} r^{-k} \mu(A \cap \{y\colon |y - x| < r\})$$

is the k-dimensional μ density of A at x; the upper and lower densities

Received February 14, 1963.