ORTHOGONAL DEVELOPMENTS OF FUNCTIONALS AND RELATED THEOREMS IN THE WIENER SPACE OF FUNCTIONS OF TWO VARIABLES[1]

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1. **Introduction.** Let C_w be the Wiener space of functions of two variables, i.e. the collection of real valued continuous functions $f(x, y)$ defined on $Q: 0 \leqq x, y \leqq 1$ and satisfying $f(0, y) = f(x, 0) = 0$. Let $F[f]$ be a complex valued functional defined almost everywhere on C_w and having Wiener measurable[1] real and imaginary parts, and let $L_2(C_w)$ be the Hilbert space of functionals $F[f]$ satisfying

$$\int_{C_w} |F[f]|^2 d_w f < \infty$$

with the inner product

$$(F_1, F_2) = \int_{C_w} F_1[f]\overline{F_2[f]} d_w f .$$

The contents of this paper are:

1. An extension of the Cameron-Martin translation theorem, Theorem III, [11].
2. An extension of the Paley-Wiener theorem to C_w. Our proof is different from that of Paley and Wiener given for the Wiener space of functions of one variable and is based on the extended Cameron-Martin translation theorem, and
3. Construction of complete orthonormal systems in $L_2(C_w)$.

2. THEOREM I[2]. *Let $p(x, y)$ be of bounded variation[3] on Q, $p(0, y)$, $p(1, y)$, $p(x, 0)$, $p(x, 1)$ be of bounded variation on the respective unit interval. Let $p(x, y)$ be continuous a.e. and bounded on Q. Let*

$$q(x, y) = \int_{Q_{xy}} p(s, t) ds\, dt \text{ where } Q_{xy} = [0, x] \times [0, y] , \quad 0 \leqq x, y \leqq 1 . \tag{2.1}$$

Let $\Gamma \subset C_w$ be Wiener measurable and a translation T be defined by

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[1] For the definition of the Wiener measure and Wiener measurable functionals, see [10] or [11].

[2] Theorem 1 can also be derived from Theorem 3, [9].

[3] For functions of bounded variation in n variables and Riemann-Stieltjes integrals with respect to them, see [11].