CONCERNING HOMOGENEITY IN TOTALLY ORDERED, CONNECTED TOPOLOGICAL SPACE

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Throughout this paper suppose that L denotes a connected, totally ordered topological space in which there is no first or last point, and whose topology is that induced by the order.

A topological space S is said to be homogeneous provided it is true that if $(x, y) \in S \times S$, there is a homeomorphism f from S onto S such that $f(x) = y$. Let H denote the set of all homeomorphisms from L onto L, and let I denote the set of all homeomorphisms which map a closed interval of L onto a closed interval of L. Let $H_0(I_0)$ denote the set of all elements of $H(I)$ which preserve order.

THEOREM 1. *If L is homogeneous, then L satisfies the first axiom of countability.*

Proof. It suffices to show that for some point z of L there exists an increasing sequence $x_1, x_2, \cdots$ and a decreasing sequence $y_1, y_2, \cdots$ such that each of these sequences converges to z. Suppose there is no such point. Let $P_1, P_2, \cdots$ denote an increasing sequence which converges to a point P and $Q_1, Q_2, \cdots$ a decreasing sequence which converges to a point Q. There is an element g in H such that $g(P) = Q$. In view of the preceding supposition, g is order reversing. There is a point R such that $g(R) = R$, and R is the limit of a sequence $R_1, R_2, \cdots$ which is either increasing or decreasing. Suppose the sequence is decreasing. The sequence $g(R_1), g(R_2), \cdots$ is increasing and converges to R. This yields a contradiction. The case where $R_1, R_2, \cdots$ is increasing is similar.

THEOREM 2. *The space L is homogeneous if and only if each pair of closed subintervals of L are topologicaly equivalent.*

Proof. Part 1. Suppose each pair of closed subintervals of L are topologically equivalent and $(x, y) \in L \times L$. There exist elements z and w of L such that $z < x < w$ and $z < y < w$, and an element g of I from $[z, x]$ onto $[z, y]$. If g is order reversing there is an element g' of I_0 from $[z, x]$ onto $[z, y]$ which may be constructed as follows: Let t denote the point of $[z, x]$ such that $g(t) = t$. g' is defined by

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