REFLECTION OF BIHARMONIC FUNCTIONS ACROSS ANALYTIC BOUNDARY CONDITIONS WITH EXAMPLES

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1. Introduction. This paper is concerned with the reflection of solutions u of the biharmonic equation $\Delta^2 u = 0$ across an analytic arc γ when u satisfies on γ two linear analytic boundary conditions. If the coefficients are subject to proper regularity conditions, then the region into which u can be extended is dependent only on the analytic arc and the original region on which u is defined; i.e., it is dependent only on geometric quantities and therefore is what may be called "reflection in the large." The case in which the boundary conditions are nonlinear was treated in [3], but extension in that case is only local.

We consider two boundary conditions whose independence is stated by an inequality. It is shown that this inequality is satisfied automatically in the case of the first, second, and mixed fundamental boundary value problems of elasticity.

Finally, we shall give applications of the first boundary value problem, in which case the reflection is effected by quadratures, for a number of special geometrical configurations. In these cases the reflection of u can be expressed explicitly.

2. Reflection across an analytic arc. Let the open analytic arc γ be defined by the real analytic relation $F(x, y) = 0$ where, for every point (x, y) on γ, $F_x(x, y) \neq 0$ or $F_y(x, y) \neq 0$.

Let $\zeta = \xi + i\eta$ and $z = x + iy$ be two complex variables and consider:

$$g(z, \zeta) \equiv F\left[\frac{z+\zeta}{2}, \frac{z-\zeta}{2i}\right] = 0$$

where $g(z, \zeta)$ is an analytic function of (z, ζ) in a polycylindrical neighborhood of $(z_0, \bar{z}_0)$ for every z_0 on γ. For $z_0 = x_0 + iy_0$ on γ:

$$g(z_0, \bar{z}_0) = F(x_0, y_0) = 0$$

and, for $(z, \zeta) = (z_0, \bar{z}_0)$,

$$\frac{\partial g}{\partial \zeta}(z_0, \bar{z}_0) = \left(\frac{1}{2}\right)\{F_x(x_0, y_0) + iF_y(x_0, y_0)\} \neq 0 \ , \quad \frac{\partial g}{\partial z} = \overline{\left(\frac{\partial g}{\partial z}\right)} \ .$$

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