SPECTRAL PERMANENCE OF SCALAR OPERATORS

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0. **Introduction.** Let $\{\mathfrak{B}_k: k \in K\}$ be a family of Banach spaces whose intersection D is dense in $\mathfrak{B}_k$ for each $k \in K$. Suppose that $0, 1 \in K$ and that each $\mathfrak{B}_k$ satisfies the relations $\mathfrak{B}_0 \cap \mathfrak{B}_1 \subset \mathfrak{B}_k$ and $\mathfrak{B}_0 < \mathfrak{B}_k < \mathfrak{B}_1$ (see § 1); let T be a linear operator which is simultaneously defined and bounded on each member of the family $\{\mathfrak{B}_k: k \in K\}$ (that is, $T \in [\mathfrak{B}_k]$; see 0.1). This paper gives a condition insuring that $\sigma(T; [\mathfrak{B}_0]) = \sigma(T; [B_k])$ for any $k \neq 1$; here $\sigma(T; [\mathfrak{B}_k])$ denotes the spectrum of T relative to $[\mathfrak{B}_k]$. In fact, this spectral equality holds whenever T has a spectral resolution which is bounded in $[\mathfrak{B}_1]$; see § 1. In this connection, it should be mentioned that the articles of Halberg and A. E. Taylor [4, 5] study relations between $\sigma(T; [\mathfrak{B}_0])$ and $\sigma(T; [\mathfrak{B}_1])$ in the particular case $K = \{0, 1\}$.

Let $\mathscr{D}_X$ be the Banach algebra of all complex-valued functions of bounded variation on a finite interval X. Our end-result depends on the fact that an operational calculus into $[\mathfrak{B}_0]$ induces a continuous representation of some $\mathscr{D}_X$ into $[\mathfrak{B}_k]$ (for $k \neq 1$); as is the case with the "spectral distributions" of Foias [3], properties of such representations may be exploited to some extent: see § 3.

Let μ be a continuous representation of $\mathscr{D}_X$ into some Banach algebra $\mathfrak{E}$, and let Σ_m be the largest open set G such that $\mu[a, b[= O$ whenever $[a, b] \subset G$; the existence of Σ_m is established in § 2. If $h \in \mathscr{D}_X$, then $\mu(h)$ is a member T of $\mathfrak{E}$, and the spectrum $\sigma(T; \mathfrak{E})$ coincides with the image $h(\Sigma_m)$ whenever h is continuous on X. Let g be a function on $h(\Sigma_m)$; as we shall see, it is natural to write $g(T) = \mu(g \circ h)$. It will be shown that

$$\sigma(g(T); \mathfrak{E}) = g(\sigma(T; \mathfrak{E})) = (g \circ h)(\Sigma_m) \tag{1}$$

whenever the images $h(x)$ and $(g \circ h)(X)$ are plane rectifiable continuous curves: see 3.2. The first equality in (1) is well-known when g is a polynomial

$$g(\lambda) = \sum_{\nu=0}^{n} \alpha_\nu \lambda^\nu \qquad (\text{for } \lambda \in h(X));$$

note that[1], in this case

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[1] The composition $g \circ h$ is defined by the relation $(g \circ h)(\lambda) = g(h(\lambda))$ for any $\lambda \in (-\infty, \infty)$.