# ANOTHER CONFORMAL STRUCTURE ON IMMERSED SURFACES OF NEGATIVE CURVATURE

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1. Everyone is familiar with the ordinary conformal structure on oriented surfaces immersed smoothly in $E^3$. This standard structure is obtained by using the first fundamental form as metric tensor. It is possible, however, to define very different conformal structures which are still vitally connected with the geometry of a surface's immersion in $E^3$.

Consider, for instance, the conformal structure induced upon a strictly convex surface (oriented so that mean curvature $H > 0$) by using its positive definite second fundamental form as metric tensor. (See [3] and [4].) This particular structure coincides with the usual one only on spheres.

The present paper is devoted, principally, to a description of the corresponding non-standard conformal structure on oriented surfaces of negative Gaussian curvature immersed smoothly in $E^3$. This new structure is obtained by using a specific linear combination of the first and second fundamental forms as metric tensor. It will be seen that our new structure coincides with the usual one only on minimal surfaces.

Also included below is a section describing the arithmetic of certain expressions associated with the various fundamental forms on an immersed surface. These expressions become quadratic differentials whenever *any* paticular conformal structure is introduced on a surface.

The paper closes with a theorem which generalizes a well known fact about minimal surfaces. For investigations related to the material which follows, see [5].

2. Consider an oriented surface $S$ which is $C^3$ immersed in $E^3$. We may number the principal curvatures so that

$$k_1 \geqq k_2 \tag{1}$$

holds over all of $S$. For convenience of notation, lines of curvature coordinates $x$, $y$ will always be chosen so that $k_1$ is the principal curvature in the $y \equiv$ constant direction, while $k_2$ is the principal curvature in the $x \equiv$ constant direction.

We now define the function

$$H' = \frac{k_2 - k_1}{2} = -\sqrt{H^2 - K}\ .$$

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