AN EFFECTLESS CUTTING OF A VIBRATING MEMBRANE

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Let G be a multiply connected domain bounded by an outer boundary Γ_0, inner boundaries $\Gamma_1, \Gamma_2, \cdots$, and possibly some other inner boundaries $\gamma_1, \gamma_2, \cdots$. Let u be the eigenfunction corresponding to the lowest eigenvalue λ_1 of the membrane problem

$$\Delta u + \lambda_1 u = 0 \quad \text{in } G \tag{1}$$

with

$$u = 0 \quad \text{on } \Gamma_0, \Gamma_1, \cdots \tag{2}$$

$$\frac{\partial u}{\partial n} = 0 \quad \text{on } \gamma_1, \gamma_2, \cdots .$$

We shall show that there exists a cut $\tilde{\gamma}$ consisting of a finite set of analytic arcs along which $(\partial u/\partial n) = 0$ which separates any given one of the fixed holes, say Γ_1, from the outer boundary Γ_0 and the other holes $\Gamma_2, \Gamma_3, \cdots$. This means that the membrane G may be cut in two along $\tilde{\gamma}$ without lowering its lowest eigenvalue. This fact is used in the preceding paper of J. Hersch to establish an upper bound for λ_1.

We assume that $\Gamma_0, \Gamma_1, \cdots$ have continuous normals and that $\gamma_1, \gamma_2, \cdots$ are analytic. Then it is well-known that u has the following properties:

(3) (a) $u > 0$ in G, and $\dfrac{\partial u}{\partial n} < 0$ on $\Gamma_0, \Gamma_1, \cdots$.

(b) u is analytic in $G + \gamma_1 + \gamma_2 + \cdots$.

(c) u_{xx} and u_{yy} do not vanish simultaneously.

(The last property follows from (3a) and (1)).

We define G_1 to be the set of points of G from which the fall lines, i.e. the trajectories of

$$\frac{dx}{dt} = -u_x \tag{4}$$

$$\frac{dy}{dt} = -u_y$$

reach Γ_1. By property (3a) G_1 contains a neighborhood in G of Γ_1, and its exterior contains neighborhoods in G of $\Gamma_0, \Gamma_2, \cdots$. Since u_x

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