SEMIGROUPS OF MATRICES DEFINING LINKED OPERATORS WITH DIFFERENT SPECTRA

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1. Introduction. The concept of "linked operators" was introduced by A. E. Taylor and the author in [1]. This concept was originally suggested by work involving bounded linear operators on the sequence spaces l_p. For example, if the infinite matrix (t_{ij}) defines operators T_p and T_q that are bounded on l_p and l_q, respectively, then these operators are linked. The somewhat complicated general definition of linked operators is deferred until § 2 of this paper. In [1] an isolated, specific example of linked operators with different spectra was given. The purpose of this paper is to exhibit three infinite semigroups of infinite matrices (t_{ij}), with complex coefficients, such that each of their elements defines linked operators with different spectra.

In the next section we give some preliminary definitions and notation and in the final section we prove a basic lemma and our principal theorems.

2. Preliminary definitions and notation. We first give the definition of linked operators.

DEFINITION. *Let X, Y be complex linear spaces, and Z a non-void complex linear space contained in both X and Y. Let X be a Banach space X_1, Y a Banach space Y_2 under the norms n_1, n_2 respectively. Let Z be a Banach space Z_N under the norm N defined by $N(z) = \max [n_1(z), n_2(z)]$. With the usual uniform norms let T_1, T_2 be bounded linear operators on X_1, Y_2 respectively, such that $T_1 z = T_2 z \in Z$ when $z \in Z$. Operators satisfying these conditions are said to be "linked."*

Our basic notation will be as follows: If T denotes the infinite matrix (t_{ij}), with complex coefficients, then T^t will denote its transpose, and $\bar{T}$ the matrix $(\bar{t}_{ij})$, where $\bar{z}$ is the complex conjugate of z. Let T_p denote the operator defined on l_p by the matrix T, $\| T_p \|$ its norm, and $[l_p]$ the algebra of bounded linear operators on l_p. Also let $\rho(T_p)$ denote the resolvent set of T_p, consisting of all complex λ such that $\lambda I - T_p$ defines a one-to-one correspondence of l_p onto l_p; $\sigma(T_p)$ denote the spectrum of T_p, consisting of all λ not in $\rho(T_p)$; and $|\sigma(T_p)|$ the spectral radius of T_p.

The matrix (t_{ij}) is said to be "regular" in case for every convergent sequence $[\zeta_n]$, $\lim_{n\to\infty} \zeta_n = \zeta$, each of the series $\sum_{k=1}^{\infty} t_{ik}\zeta_k$ is convergent

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