A REMARK ON ANALYTICITY OF FUNCTION ALGEBRAS

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1. Let A be a closed separating subalgebra of $C(X)$, X compact, with maximal ideal space $\mathfrak{M}_A$ and Šilov boundary ∂_A. Naturally A can also be viewed as a closed subalgebra of $C(\mathfrak{M}_A)$ or $C(\partial_A)$.

Call A *analytic on* X if the vanishing of $f \in A$ on a non-void open subset of X implies $f \equiv 0$, or simply *analytic* if this holds for $X = \mathfrak{M}_A$. Recently Kenneth Hoffman asked if the analyticity of A on ∂_A implied analyticity on $\mathfrak{M}_A$; the present note is devoted to a counter-example.[1] Evidently such an example, analytic on its Šilov boundary, must be an integral domain, so our algebra is a non-analytic integral domain.

The example was suggested by, and utilizes, an interpolation theorem of Rudin and Carleson [5, 9], recently generalized by Bishop [3], which in fact permits the construction of a variety of unfamiliar tractable subalgebras of familiar algebras; consequently we shall discuss the construction in more generality than is absolutely necessary. Finally we give a slightly more complicated example which is also dirichlet.

NOTATION. $M(X)$ will denote the space of (finite complex regular Borel) measures μ on X; for such a μ, μ is orthogonal to $A (\mu \perp A)$ if $\mu(f) = \int f d\mu = 0$, f in A. And μ_F will denote the usual restriction of μ to $F \subset X$, while $f \mid F$ will be the restriction of a function f, $A \mid F$ the set $\{f \mid F : f \in A\}$. An algebra A will always be assumed to contain the constants.

2. Our construction is based on the following fact.

(2.1) *Suppose F is a closed subset of X, and $\mu_F = 0$ for all μ in $M(X)$ orthogonal to A. Then*[2]

(2.1.1) $$A \mid F = C(F) \text{ [3]}$$

(2.1.2) *if X is metric, F is a peak set of A, i.e., there is an f in*

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[1] After this note was completed, I found that analyticity of A on $\mathfrak{M}_A$ implies analyticity on ∂_A; this will appear in a subsequent paper.

[2] (2.11) is Bishop's generalization of the Rudin-Carleson result mentioned before, which applies to the special case in which A is the "disc algebra" and F a subset of measure zero of the unit circle. (2.12) will actually be avoided in the specific examples we construct.