A SUBFUNCTION APPROACH TO A BOUNDARY VALUE PROBLEM FOR ORDINARY DIFFERENTIAL EQUATIONS

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1. Introduction. Consider the ordinary, second-order differential equation

$$y'' = f(x, y, y') \tag{1.1}$$

where $f(x, y, y')$ is a real-valued function defined on the region

$$T = \{(x, y, y') \mid a \leqq x \leqq b, |y| < \infty, |y'| < \infty\} ,$$

a and b finite.

The purpose of this paper is to determine sufficient conditions which when placed on $f(x, y, y')$ guarantee the existence of a unique solution of the two-point boundary value problem (BVP):

$$y'' = f(x, y, y') , \quad y(a) = \alpha , \quad y(b) = \beta . \tag{1.2}$$

A solution of the BVP: $y'' = f(x, y, y')$, $y(x_1) = y_1$, $y(x_2) = y_2$, where $a \leqq x_1 \leqq x_2 \leqq b$ will be defined to be a function $y(x)$ which is of class C^2 and satisfies (1.1) on (x_1, x_2), which is continuous on $[x_1, x_2]$, and which assumes the given boundary values at x_1 and x_2.

The following assumptions will be placed on $f(x, y, y')$ as needed.

(A_0) $f(x, y, y')$ is continuous on T.

(A_1) $f(x, y, y')$ is a non-decreasing function of y for each fixed x and y' in T.

(A_2) $f(x, y, y')$ satisfies a Lipschitz condition with respect to y' on each fixed compact subset to T.

The primary results of this paper are the following two theorems.

THEOREM 6.2. *If*

(1) $f(x, y, y')$ *satisfies* A_0, A_1, *and* A_2,

(2) *there exists a positive continuous function* $\phi(u)$ *defined for* $u \geqq 0$ *such that*

$$|f(x, y, y') - f(x, y, 0)| \leqq K_s\phi(|y'|)$$

where K_s *is a constant depending on compact subsets* S *of*

$$\{(x, y) \mid a \leqq y \leqq b, |y| < \infty\} , \quad (x, y) \in S , \quad |y'| < \infty ,$$

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