ON SOME PROPERTIES OF SOLUTIONS OF $\Delta\psi + A(r^2)X\nabla\psi + C(r^2)\psi = 0$

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Dedicated to Charles Loewner on his 70th birthday

Introduction. The main task of Bergman's operator theory has been to establish and to study the mappings (in the large) of the algebra $\{f\}$ of functions of one or several complex variables onto the linear space $\{\psi\}$ of solutions of various classes of linear partial differential equations $L[\psi] = 0$. In the case of partial differential equations of two variables the mappings of the algebra of functions of one complex variable onto the solutions $\{\psi\}$ of a general class of equations is established by means of a comparatively simple integral operator in such a way that very many theorems concerning $\{f\}$ have their counterpart in $\{\psi\}$.

In the case of partial differential equations of three variables the mapping of the set of functions $\{f\}$ of two complex variables onto the space $\{G\}$ of harmonic function and onto the space $\{\psi\}$ of solutions of the equation

$$(1) \qquad \Delta\psi(x, y, z) + A(r^2)\, X\nabla\psi(x, y, z) + C(r^2)\,\psi(x, y, z) = 0$$

where $r^2 = x^2 + y^2 + z^2$, $X = (x, y, z)$, Δ is Laplace operator,

$$X\nabla = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z}$$

and $A, C,$ are entire functions of r^2, in particular have been studied ([1], [4]).

In addition one can consider the mapping $\{G\} \to \{\psi\}$. Of course, this mapping could be obtained by combining $\{f\} \to \{G\}$ and $\{f\} \to \{\psi\}$. However, in this way one obtains a very complicated integral operator and since we are passing through functions of two complex variables which then have to be restricted to some special values, various relations could be lost. Therefore, it is of interest to study the direct mapping $\{G\} \to \{\psi\}$ which is the aim of the present paper.

Let

$$a_n(r) = \frac{\Gamma\left(n + \frac{1}{2}\right)}{\Gamma(n)\,\Gamma\left(\frac{1}{2}\right)}\, C_n(r)$$

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