ON THE CONVERGENCE OF SEMI-DISCRETE ANALYTIC FUNCTIONS

G. J. KUROWSKI

1. Introduction. In a previous paper [3], the author has presented the basic concepts and definitions for semi-discrete analytic functions. These functions are defined on two types of semi-lattices (sets of lines in the xy-plane, parallel to the x-axis)—one of which leads to a symmetric theory. We will concern ourselves here only with the symmetric case. These functions satisfy the following defining equation [3] on a region of the semi-lattice

$$(1.1) \qquad \frac{\partial f(z)}{\partial x} = [f(z + ih/2) - f(z - ih/2)]/ih ,$$

where $h > 0$ is the spacing of the semi-lattice. For convenience, we will repeat the definition of the symmetric semi-lattice and its associated odd and even semi-lattices. A grid-line, a_m, is the set of points in the xy-plane such that $y = mh$ where $h > 0$. The union $G(2k, h)$ of the a_m for $m = k$ $(k = 0, \pm 1, \pm 2, \cdots)$ is called the *even* semi-lattice; the union $G(2k + 1, h)$ of the a_m for $m = (2k + 1)/2$ is called the *odd* semi-lattice. The semi-discrete z-plane is the union of $G(2k, h)$ and $G(2k + 1, h)$. It will be denoted by $L(h)$. Additional concepts such as domains, paths, path-integrals, etc., are defined in [3]. The following notational conventions will be employed:

$$(1.2) \qquad f_k = f(x + ihk) = f_k(x) ,$$

and the abbreviation SD will be used to stand for semi-discrete.

2. Sub and super harmonic semi-discrete functions. In the continuous case, it is well-known that if a function $u(x, y)$ is defined over a region R of the plane and if, further, $\Delta^2(u) \geqq 0$ for all $(x, y) \in R$, where Δ^2 denotes the two dimensional Laplacian; then $u(x, y)$ cannot have a maximum on the interior of R. Such a function is said to be *sub-harmonic* in R [2]. Similarly, if the function $u(x, y)$ defined on R satisfies the equation $\Delta^2(u) \leqq 0$ for all $(x, y) \in R$; then $u(x, y)$ cannot have a minimum on the interior of R. Such a function is said to be *super-harmonic* in R [2]. An analogous result holds for semi-discrete functions which are defined on domains of either the even or odd semi-lattice. To be specific, we will consider functions $u(x, y)$ defined on

Received April 17, 1963. Duke University Research Associate, "Special Research in Numerical Analysis," sponsored by the Army Research Office (Durham), U.S. Army, Contract DA-31-124-AROD-13.